

# Dynamic Urban Economics

Greaney, Parkhomenko, & Van Nieuwerburgh (2025)

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  1. people and places are **heterogeneous**,
  2. people find it **costly to move** across places, and
  3. some people own large, immobile, undiversified assets—**homes!**

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*Ambitious paper with many moving parts!*

# Solving dynamic spatial models is hard

**Desmet and Rossi-Hansberg (2014)** pointed out two key challenges...

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... to which I'll add two more:

3. usually requires thinking about **individual heterogeneity**, not just locational
4. number of parameters to estimate generally **scales with number of locations**, but model inversion unlikely to work w/ heterogeneity

# The dimensionality at hand

- **stationary equilibrium:**
  - **individual states:** age  $a$ , liquid wealth  $b$ , house size  $h$ , labor productivity  $\zeta$ , residential location  $i$ , workplace location  $j$
  - **market clearing:** sufficient to solve for  $\{L_{it}, r_{Sit}\}$
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Ultimately, encode model as **state-space transition matrix**...

...so prefer a **smaller, sparser, or more-structured** encoding!

# So how do we make progress? A “mixed time” approach

1. **cast the model in continuous time**. . . (Achdou et al., 2022)
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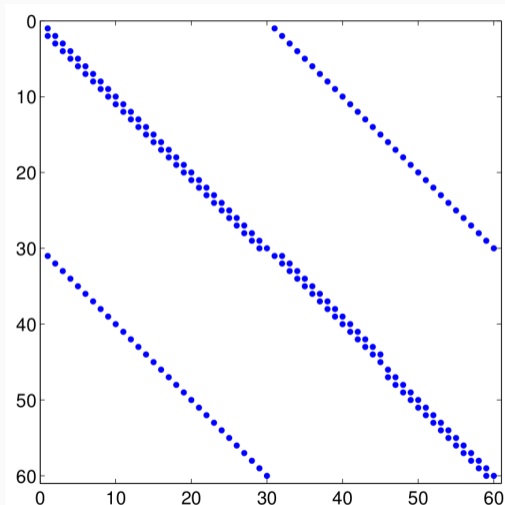
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2. ... **but discrete shocks/choices at fixed “shock ages”** ([Greaney, 2023](#))
3. ... **and keep small what we can**
  - 5 grid points for labor productivity (Rouwenhorst)
  - 6 grid points for house size (total floorsize, not unit size, is stock)
  - 50 grid points for liquid wealth
  - current wage is sufficient stat. for workplace choice (more on this later)

## Mixed time, visualized

Zoom in on **residential location choice**...

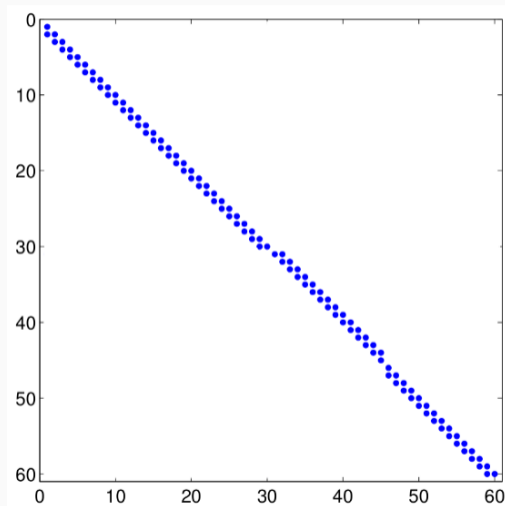
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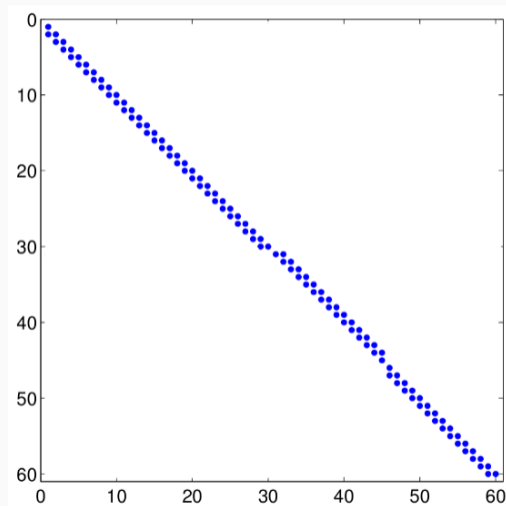


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... all of  $\{\zeta, h, i, j\}$  operate like this!





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- **clarity:** I grok how HJB works but—even still—KFE **at vs. between** shock ages?
- **possibly-distorted decisions:**
  - are there any **turnpike-like dynamics** near shock ages?
    - different behavior for “I know my shock age is in 5 years” vs. “A shock can happen whenever, but I expect it in 5 years”?
    - presumably depends on relative size of shock age vs. time step
    - computation seems to use 1-for-1, so it’s always a shock age?
  - timing of **workplace location choice** buys a lot...
    - at shock age,  $j$  freely chosen + amenities enjoyed + commuting costs paid...
    - ...so  $j$  doesn’t matter *between* shock ages except through  $w_j \rightarrow$  can reduce  $N^2$  to  $N \times |w|$
- ...**but it seems brittle:**
  - what if we think people choose  $j$  and  $i$  jointly? or just  $j$  first?
  - what if we add human capital so wage growth depends on  $j$ ?
  - already, have to treat wages as **constant** between shock ages along transition path (fn.16)

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- **speed?** What's the fully-continuous time version? How slow would it be?
- **insight?** Counterfactuals without homeownership or consumption-saving. . .
  - make the **same positive predictions** → can recalibrate with fewer moments to get same response (high migration cost vs. mid migration cost + house transaction cost)
  - welfare effects are **less dispersed** somewhat by construction → we eliminated margins of heterogeneity!
- **predictive power?** Would prefer different counterfactuals. . .
  - see pre- and post-shock data ([Dingel and Tintelnot, 2021](#))
  - show static or reduced dynamic model, calibrated to same moments, does **worse** than the full model

# How easily can we generalize?

If I want to use this approach in my work, but I want to add...

- **additional states:** shock age or not?
- **additional locations:** showed scaling for PE stationary, but estimation? transitions?
- **local externalities:** what about **uniqueness of transition path?**
  - guess smoothly-decaying weighted avg. between old and new S.S. values...
  - ...but with externalities, could there be multiple paths to same S.S.?
- **open city:** not important for tractability, but closed-city could be driving welfare
  - decompose welfare gain into  $\Delta(\text{house value})$  and  $\Delta(\text{labor market})$

Thank you!

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