A Unified Urban Model with Non-Homothetic Housing Demand

Robert-Nicoud, Combes, Duranton, & Gobillon (2026)

Levi Crews (UCLA)

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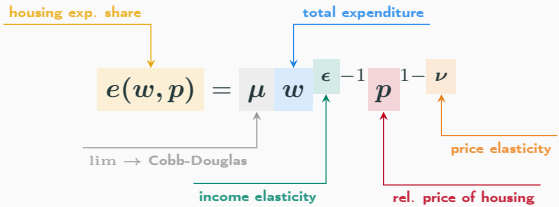
PIGL–UM = A *simple* model in the highest sense. . .

1. identical HH with PIGL pref. (ϵ , ν)



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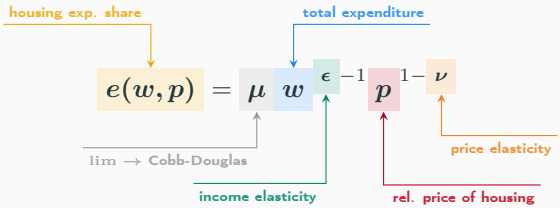
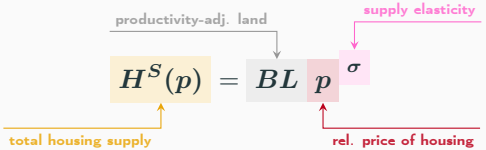


2. **competitive landlords use Cobb-Douglas on fixed land L with share $1/(1+\sigma)$**



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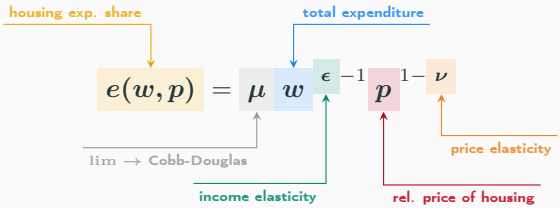

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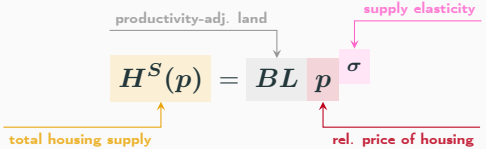

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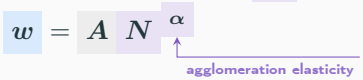

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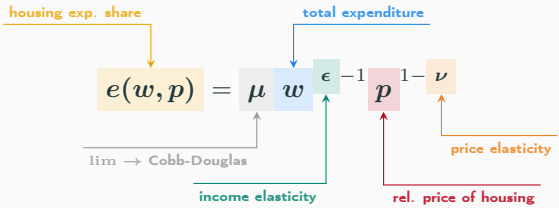

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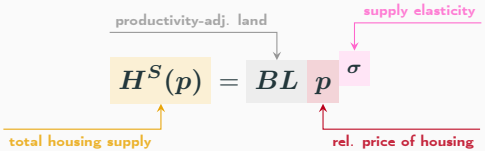

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$$w = A\, N^{\alpha}$$

agglomeration elasticity

$\to$ leads to a closed-form solution for the equilibrium:

$$p = \left(\frac{\mu\, N}{BL}\right)^{\frac{1}{\nu+\sigma}} w^{\frac{\epsilon}{\nu+\sigma}} \qquad H = \left(BL\right)^{\frac{\nu}{\nu+\sigma}} \left(\mu\, N\right)^{\frac{\sigma}{\nu+\sigma}} w^{\frac{\sigma\,\epsilon}{\nu+\sigma}}$$

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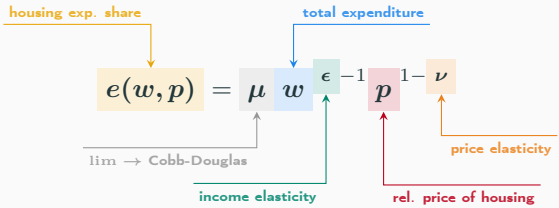

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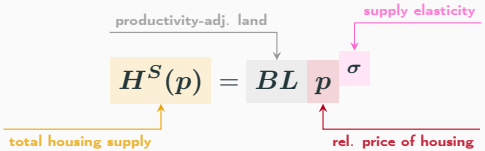

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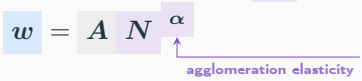

$\to$ leads to a closed-form solution for the equilibrium:

$$p \propto N^{\frac{1+\alpha\,\epsilon}{\nu+\sigma}} \qquad H \propto N^{\frac{\sigma\left(1+\alpha\,\epsilon\right)}{\nu+\sigma}}$$

. . . and the culmination of 20+ years of theory ↔ data work

Elasticity	Notation	Estimates	Citation
Income (demand)	ϵ	**0.7–0.9**	**Combes, Duranton, and Gobillon (2019, 2021)**, Finlay and Williams (2025)
Price (demand)	ν	**0.6–0.9**	**Combes, Duranton, and Gobillon (2019, 2021)**, Finlay and Williams (2025)
Price (supply)	σ	**2 (0–10)**	**Combes, Duranton, and Gobillon (2019); Baum-Snow and Duranton (2025)**
Agglomeration	α	**0.02–0.05**	**Combes, Duranton, and Gobillon (2008); Combes and Gobillon (2015)**
Migration	κ	**2.1–5.1**	Beaudry, Green, and Sand (2014); Diamond (2016); Finlay and Williams (2025)
Commuting	θ	**0.04–0.08**	**Combes, Duranton, and Gobillon (2019); Duranton and Puga (2023)**

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2. **even those who specify NHCES actually *estimate* PIGL** (Finlay and Williams, 2025)

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can I use PIGL–UM to rationalize time trends?

Requiem or revival for Davis and Ortalo-Magné (2011)?

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- ~~**Preferences for housing are always and everywhere Cobb-Douglas**~~ $\not\Rightarrow$
  - actually slightly *increasing* across cities
    (**Combes, Duranton, and Gobillon, 2019**; Finlay and Williams, 2025)
  - can match with unit housing demand
    (**Behrens, Duranton, and Robert-Nicoud, 2014**; Davis and Dingel, 2019)

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- **What if we add constant-over-time $e(w_{nt}, p_{nt})$ to our list of requirements?**

The simplest back-of-the-envelope

If I assume that A *grows at rate* g_A ...

$$e(AN^{\alpha}, p) = \mu \left(A \, N^{\alpha} \right)^{\epsilon - 1} p^{1 - \nu}$$

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Then to hold expenditure shares constant over time,

$$0 = (\epsilon - 1) \left(g_A + \alpha g_N \right) + (1 - \nu) \, g_p$$

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$$g_p = \frac{1 - 0.75}{1 - 0.61} \left(g_A + \alpha g_N \right)$$

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$$g_p \approx 0.64 \left(g_A + \alpha g_N \right)$$

$\rightarrow$ **Need relative price growth *slower* than aggregate income growth!**

Potpourri for the authors

- non-homotheticities matter for restoring finite efficient city size, but...
  - **parameter condition doesn't depend on how strong they are**: just $\alpha \ \sigma < 1$
  - vestige of the simplifying assumption that $\epsilon = \nu \in (0, 1)$?
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- PIGL–AMM assumes $p(x)\tau x^{\theta}$, not $p(x) + \tau x^{\theta} \rightarrow$ consistent with estimation of θ? **(Combes, Duranton, and Gobillon, 2019; Duranton and Puga, 2023)**
  - can have non-homothetic housing (unit demand) but homothetic urban costs (Crews, 2024)

Recap

A parsimonious model of urban structure with non-homothetic housing demand

1. should be taught in any urban/spatial course as a unifying framework
2. PIGL is a smart choice $\longrightarrow$ **make it clearer *why***
3. non-homothetic preferences $\longrightarrow$ ***can we still divorce from time trends?***

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Thanks!

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