Some of the following problems are adapted from Jones and Vollrath (2024, Ch.2). To practice for the exams, **please show your work.**

1. **TFUs.** Determine whether the following statements are **true**, **false**, or **uncertain**, and justify your answer in no more than one paragraph. Please be concise.[1]

   (a) In steady state, the growth rate of the capital stock (g_K) is equal to the growth rate of GDP (g_Y).

   (b) A steady state of the Solow model exists for any value of $\alpha \in [0, 1]$.

   (c) Along a balanced growth path, the savings rate (s) affects both the *level* and the *growth rate* of GDP per capita.

2. **Comparative statics.** For each of the following scenarios, assume that the economy begins in a steady state. For each scenario, draw three figures showing how each of the following terms evolve over time: the amount of capital per efficiency unit of labor ($\tilde{k}_t$), the growth rate of GDP per capita (g_y), and the log of GDP per capita ($\ln(y_t)$). Be sure to clearly label all axes and curves in your figures.

   (a) The savings rate (s) drops to a lower value and stays there permanently.

   (b) The growth rate of productivity (g_A) rises to a higher value and stays there permanently.

   (c) There is a one-time decrease in the size of the capital stock (K).

3. **Harrod- vs. Hicks-neutral productivity.** The goal of this problem is to demonstrate that the Solow model with a Cobb-Douglas production function can also be solved if productivity is specified as Hicks-neutral rather than Harrod-neutral.

   (a) Consider two production functions:

   $$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \quad \text{(Harrod-neutral)}$$
   $$Y_t = B_t K_t^{\alpha} L_t^{1-\alpha} \quad \text{(Hicks-neutral)}$$

   What must be the relationship between A_t and B_t for these two production functions to yield the same output for the same inputs at all t? What must be the relationship between g_A and g_B?

   (b) Suppose that firms in the economy use the Hicks-neutral production function. What is the rental rate they pay for capital (r_t)? What is the wage they pay for labor (w_t)?

   (c) Let $k_t \equiv K_t/L_t$ be capital per worker. Derive an expression for GDP per capita ($y_t = Y_t/L_t$) in terms of k_t and B_t.

   (d) Derive an expression for the growth rate of GDP per capita (g_y) in terms of α, g_K, g_L, and g_B. Is this consistent with your answer to part (a) and the expression we derived in class for the Harrod-neutral case?

   (e) Using the capital accumulation equation, try to derive an expression for the growth rate of capital per worker, g_k, in terms of only k_t, α, s, δ, g_B, and g_L. What's the problem?

[1] On an exam, your score would be based almost entirely on the justification you provide, not on the specific **TFU** designation you choose.

(f) Using your answer from part (a), derive an expression for capital per efficiency unit of labor ($\tilde{k}_t$) in terms of the Hicks-neutral productivity (B_t). Then derive an expression for its growth rate, $g_{\tilde{k}}$, in terms of only $\tilde{k}_t$, α, s, δ, g_B, and g_L.

(g) Using that expression, solve for steady state $\tilde{k}^{\text{ss}}$.

(h) Along the balanced growth path, what's the growth rate of GDP per capita in terms of g_B? Now, using your answer from part (a), write it in terms of g_A. Is this consistent with the Harrod-neutral case?

Jones, Charles I. and Dietrich Vollrath. 2024. *Introduction to economic growth*. New York: W.W. Norton & Company, fourth ed.