To practice for the exams, **please show your work.**

1. **TFUs.** Determine whether the following statements are **true**, **false**, or **uncertain**, and justify your answer in no more than one paragraph. Please be concise.[1]

   (a) Let $\{\bar{c}_t\} \equiv \{c_t \,|\, c_t = \bar{c}, \forall t\}$ be a constant consumption path for which the corresponding steady-state capital stock $\bar{k}^{\text{ss}}$ exists and is strictly greater than the golden rule level k^{GR}. Then there exists an alternative, feasible consumption path $\{\hat{c}_t\}$ that a representative household would strictly prefer in every period.

   (b) A steady state of the neoclassical growth model exists for any rate of depreciation $\delta \in [0, 1]$.

   (c) All else equal, the transition to steady state happens more quickly in the Ramsey-Kass-Coopmans model when the household's discount factor β is closer to one.

2. **Comparative statics.** For each scenario, draw the c-k phase diagram to show how consumption and the capital stock evolve over time. Be sure to clearly label all axes, curves, and intersection points in your figure. Use arrows to indicate how capital and consumption would move in each region of the diagram.

   (a) Assume that the economy begins in a steady state. The rate of capital gains tax rises from zero to $\tau > 0$ in every period, so the household's per-period budget constraint becomes

   $$c_t + a_{t+1} = [1 + r_t(1 - \tau)]a_t + w_t.$$

   (b) Assume that the economy begins in a steady state. There is a one-time decrease in the size of the capital stock (k).

   (c) Assume that the economy begins at the golden rule capital stock. There is no change in the economy's parameters or initial conditions.

3. **The Brock and Mirman (1972) model.** The goal of this problem is to solve a special case of the Ramsey-Cass-Koopmans model in closed form. Throughout, we'll assume that $L_t = 1$ (no population growth), $A_t = 1$ (no productivity growth), $\delta = 1$ (full depreciation), the representative firm uses a Cobb-Douglas production function,

   $$y_t = F(k_t, \ell_t) = k_t^{\alpha} \ell_t^{1-\alpha},$$

   and the representative household has log utility,

   $$U(c_t) = \ln c_t.$$

   The household per-period budget constraint is

   $$c_t + a_{t+1} = (1 + r_t)a_t + w_t,$$

   where a_t is the household's stock of assets at the beginning of period t, r_t is the *time-varying* interest rate, and w_t is the wage rate.

   (a) Derive the household's lifetime budget constraint.

[1] On an exam, your score would be based almost entirely on the justification you provide, not on the specific **TFU** designation you choose.

(b) Set up the Lagrangian for the household's optimization problem.

(c) Characterize the household's optimal consumption path in closed form in terms of prices ($\{w_t, r_t\}$), initial assets (a_0), and parameters.

(d) Solve the firm's profit maximization problem.

(e) In equilibrium, what is the value of ℓ_t? What is the relationship between k_t and a_t? Why?

(f) Recall the resource constraint,

$$k_{t+1} = F(k_t, 1) - c_t + (1 - \delta)k_t.$$

Use the resource constraint and your answers to (c)–(e) to characterize the steady state of the economy in terms of k^{ss}, c^{ss}, and parameters. For what savings rate, if any, does the economy behave like the Solow model in steady state?

(g) Draw the c-k phase diagram that characterizes the transitional dynamics of the economy. Be sure to clearly label all axes, curves, and intersection points in your figure. Use arrows to indicate how capital and consumption would move in each region of the diagram.

(h) Mark the golden rule level of capital and consumption on your diagram. How does it compare to the steady state you characterized in (f)? Why? What happens as $\beta \to 1$?

Brock, W. and L. Mirman. 1972. "Optimal growth under uncertainty: The discounted case." *Journal of Economic Theory* 4:479–513.