To practice for the exams, **please show your work.**

1. **TFUs.** Determine whether the following statements are **true**, **false**, or **uncertain**, and justify your answer in no more than one paragraph. Please be concise.[1]

   (a) The pure externality model with $B_t = AK_t^\eta$ generates positive, steady-state growth for any value of $\eta \in [0, 1]$.

   (b) For any values of λ and ϕ, the ratio L_t/A_t is constant in a steady state of the Romer model.

   (c) The interest rate in the Romer model is strictly lower than the interest rate in the Solow model for the same capital/output ratio. (*Hint*: First, what *is* the interest rate in the Romer model?)

2. **Comparative statics.** For each of the following scenarios, assume that the economy begins in a steady state with $\lambda, \phi < 1$. For each scenario, draw three figures showing how each of the following terms evolve over time: the growth rate of productivity (g_A), the log of productivity ($\ln A_t$), and the log of GDP per capita ($\ln y_t$). Be sure to clearly label all axes and curves in your figures. (*Hint*: Start by drawing a phase diagram like Week 6 slide 17.)

   (a) Innovation becomes easier, meaning θ rises to a higher value and stays there permanently.

   (b) The allocation of workers to research, s_R, drops to a lower value and stays there permanently.

   (c) The population growth rate, g_L, rises to a higher value and stays there permanently.

3. **The Uzawa (1965)-Lucas (1988) model.** The goal of this problem is to solve the Uzawa-Lucas model (Week 5 slides 8–9). Throughout, we'll assume that Hicks-neutral productivity A is constant, the representative firm uses a Cobb-Douglas production function,

$$Y_t = AK_t^\alpha (uh_tL_t)^{1-\alpha}$$

   where u is the exogenous share of labor used for production, physical capital accumulation obeys

$$\dot{K}_t = sY_t - \delta K_t,$$

   and human capital accumulation obeys

$$\dot{h}_t = (1-u)h_t.$$

   (a) Define capital per skill-adjusted worker, $\hat{k}_t = K_t/(uh_tL_t)$. Write the production function for GDP per capita, y_t, in terms of $\hat{k}_t$.

   (b) Derive an expression for g_K in terms of $\hat{k}_t$ using the resource constraint for physical capital.

   (c) Decompose the growth rate of $\hat{k}_t$ in terms of g_K, g_L, and u.

   (d) What must be true of the growth rate of $\hat{k}_t$ in steady state?

[1] On an exam, your score would be based almost entirely on the justification you provide, not on the specific **TFU** designation you choose.

(e) Solve for the steady-state value of $\hat{k}_t$. (Hint: use a phase diagram.)

(f) Derive an expression for g_y at any time t. What is it along the BGP?

(g) Derive an equation for $\ln y_t$ along the BGP.

(h) For each of s and u, state whether it affects the steady-state *level* of GDP per capita, the steady-state *growth rate* of GDP per capita, or *both*. Explain.

Lucas, Robert E., Jr. 1988. "On the mechanics of economic development." *Journal of Monetary Economics* 22 (1):3–42.

Uzawa, Hirofumi. 1965. "Optimum technical change in an aggregative model of economic growth." *International Economic Review* 6 (1):18–31.