

ECON 164: Theory of Economic Growth

Week 3: The Neoclassical Growth Model

Levi Crews

Winter 2026

Recap

- Last time, we developed the **Solow model**, which has two building blocks:
 - the aggregate production function:

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$

- **endogenous** capital accumulation...

$$\dot{K}_t = sY_t - \delta K_t$$

Recap

- Last time, we developed the **Solow model**, which has two building blocks:
 - the aggregate production function:

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$

- **endogenous** capital accumulation w/ **exogenous saving rate**:

$$\dot{K}_t = s Y_t - \delta K_t$$

Recap

- Last time, we developed the **Solow model**, which has two building blocks:
 - the aggregate production function:

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$

- **endogenous** capital accumulation w/ **exogenous saving rate**:

$$\dot{K}_t = s Y_t - \delta K_t$$

- But this picture is incomplete: *what determines variation in s ?*

Recap

- Last time, we developed the **Solow model**, which has two building blocks:
 - the aggregate production function:

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$

- **endogenous** capital accumulation w/ **exogenous saving rate**:

$$\dot{K}_t = s Y_t - \delta K_t$$

- But this picture is incomplete: *what determines variation in s ?*
- This week: Solow w/ **endogenous** saving \rightarrow Ramsey-Cass-Koopmans model

(in discrete time, h/t **Adrien Bila**)

What are we saving *for*?

- Endogenizing saving behavior requires taking a stance on what we're saving for
- Most of us save to **spend** more later!
- This is why we'll be analyzing **consumption and saving *jointly***

What should go into a model of consumption/saving?

- Solow model: consumption/saving only depend on **current income Y_t**

$$C_t = (1 - s) \cdot \mathbf{Y}_t, \quad I_t = s \cdot \mathbf{Y}_t$$

What should go into a model of consumption/saving?

- Solow model: consumption/saving only depend on **current income Y_t**

$$C_t = (1 - s) \cdot \mathbf{Y}_t, \quad I_t = s \cdot \mathbf{Y}_t$$

- What else should C_t and I_t depend on other than \mathbf{Y}_t ?

What should go into a model of consumption/saving?

- Solow model: consumption/saving only depend on **current income Y_t**

$$C_t = (1 - s) \cdot \mathbf{Y}_t, \quad I_t = s \cdot \mathbf{Y}_t$$

- What else should C_t and I_t depend on other than \mathbf{Y}_t ?

- future Y_t ? age/life cycle?
- past Y_t ? wealth?
- impatience?
- interest rates? taxes?
- uncertainty?
- borrowing constraints?

What should go into a model of consumption/saving?

- Solow model: consumption/saving only depend on **current income Y_t**

$$C_t = (1 - s) \cdot \mathbf{Y}_t, \quad I_t = s \cdot \mathbf{Y}_t$$

- What else should C_t and I_t depend on other than \mathbf{Y}_t ?

- future Y_t ? age/life cycle?
- past Y_t ? wealth?
- impatience?
- interest rates? taxes?
- uncertainty?
- borrowing constraints?

- Take *microfounded* approach: saving is **chosen** by a rational utility-maximizing rep. HH
- **This week:** in *discrete* time...
 - two periods (ECON 102)
 - ∞ periods
 - ***back to growth!***

Two-period consumption-saving

Utility function

Consider the behavior of a **representative household** over two periods ($t = 1, 2$)

Utility function

Consider the behavior of a **representative household** over two periods ($t = 1, 2$)

- $U(C) \equiv$ **per-period (flow)** utility of the household over consumption

Utility function

Consider the behavior of a **representative household** over two periods ($t = 1, 2$)

- $U(C) \equiv$ **per-period (flow)** utility of the household over consumption
- $U'(C) \equiv$ **marginal** utility
 - assume *positive* but *decreasing* ($\rightarrow U$ is strictly concave) *[interpretation?]*
 - e.g., if $U(C) = \ln C$ then $U'(C) = \frac{1}{C}$

Utility function

Consider the behavior of a **representative household** over two periods ($t = 1, 2$)

- $U(C) \equiv$ **per-period (flow)** utility of the household over consumption
- $U'(C) \equiv$ **marginal** utility
 - assume *positive* but *decreasing* ($\rightarrow U$ is strictly concave) [interpretation?]
 - e.g., if $U(C) = \ln C$ then $U'(C) = \frac{1}{C}$
- $U(C_1) + \beta U(C_2) \equiv$ **lifetime utility**, where $\beta \in (0, 1)$ is the discount factor

Budget constraint

Suppose the household receives income Y_t at date t for $t = 1, 2$

- saving S earns interest rate r
- the **per-period (flow)** budget constraints:

$$C_1 + S = Y_1$$

$$C_2 = Y_2 + (1 + r)S$$

- How to turn these *two* constraints into a single one?

Present discounted value

The **present discounted value (PDV)** of a payment D at date t ...

how much you need to save at date 0 to have D at date t

Present discounted value

The **present discounted value (PDV)** of a payment D at date t ...

how much you need to save at date 0 to have D at date t

- Examples:
 - To have \$1 at $t = 1$, you need to save $\text{PDV} = \frac{1}{1+r}$ at $t = 0$
 - To have \$1 at $t = 2$, you need to save $\text{PDV} = \frac{1}{(1+r)^2}$ at $t = 0$

Present discounted value

The **present discounted value (PDV)** of a payment D at date t ...

how much you need to save at date 0 to have D at date t

- Examples:
 - To have \$1 at $t = 1$, you need to save $\text{PDV} = \frac{1}{1+r}$ at $t = 0$
 - To have \$1 at $t = 2$, you need to save $\text{PDV} = \frac{1}{(1+r)^2}$ at $t = 0$
- What if you get \$1 each year from this year onward?

$$\text{PDV} = 1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \dots = \underbrace{\sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i}_{\text{geometric sum}} = \frac{1}{1 - \frac{1}{1+r}} = \frac{1+r}{r}$$

The lifetime budget constraint

Combine the per-period budget constraints into a single **lifetime budget constraint**:

1. Divide the second period constraint by $(1 + r)$:

$$C_2 = Y_2 + (1 + r)S \quad \rightarrow \quad \frac{C_2}{1 + r} = \frac{Y_2}{1 + r} + S$$

2. Substitute in $S = Y_1 - C_1$:

$$\underbrace{C_1 + \frac{C_2}{1 + r}}_{\text{PDV of consumption}} = \underbrace{Y_1 + \frac{Y_2}{1 + r}}_{\text{PDV of income}}$$

The lifetime budget constraint

Combine the per-period budget constraints into a single **lifetime budget constraint**:

1. Divide the second period constraint by $(1 + r)$:

$$C_2 = Y_2 + (1 + r)S \quad \rightarrow \quad \frac{C_2}{1 + r} = \frac{Y_2}{1 + r} + S$$

2. Substitute in $S = Y_1 - C_1$:

$$\underbrace{C_1 + \frac{C_2}{1 + r}}_{\text{PDV of consumption}} = \underbrace{Y_1 + \frac{Y_2}{1 + r}}_{\text{PDV of income}}$$

Coming up: *this holds more generally!*

Two formulations of the two-period consumption-saving problem

1. The **per-period** formulation:

$$\begin{aligned} \max_{C_1, C_2, S} \quad & U(C_1) + \beta U(C_2) \quad \text{s.t.} \quad C_1 + S = Y_1 \\ & C_2 = Y_2 + (1 + r)S \end{aligned}$$

2. The **lifetime** formulation:

$$\max_{C_1, C_2} \quad U(C_1) + \beta U(C_2) \quad \text{s.t.} \quad C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}$$

Two ways to solve

Substitution → *unconstrained optimization*
using per-period budget constraints...

$$\begin{aligned} \max_S \quad & U(Y_1 - S) + \beta U(Y_2 + (1 + r)S) \\ \rightarrow \quad & U'(Y_1 - S) = \beta(1 + r)U'(Y_2 + (1 + r)S) \end{aligned}$$

Two ways to solve

Substitution → unconstrained optimization using per-period budget constraints...

$$\begin{aligned} \max_S \quad & U(Y_1 - S) + \beta U(Y_2 + (1+r)S) \\ \rightarrow \quad & U'(Y_1 - S) = \beta(1+r)U'(Y_2 + (1+r)S) \end{aligned}$$

Lagrangian = constrained optimization using lifetime budget constraint...

$$\begin{aligned} \max_{C_1, C_2} \quad & U(C_1) + \beta U(C_2) \\ & + \lambda \left(Y_1 + \frac{Y_2}{1+r} - C_1 - \frac{C_2}{1+r} \right) \\ \rightarrow \quad & U'(C_1) = \lambda, \quad \beta U'(C_2) = \frac{\lambda}{1+r} \end{aligned}$$

Two ways to solve

Substitution → unconstrained optimization using per-period budget constraints...

$$\begin{aligned} \max_S \quad & U(Y_1 - S) + \beta U(Y_2 + (1+r)S) \\ \rightarrow \quad & U'(Y_1 - S) = \beta(1+r)U'(Y_2 + (1+r)S) \end{aligned}$$

Lagrangian = constrained optimization using lifetime budget constraint...

$$\begin{aligned} \max_{C_1, C_2} \quad & U(C_1) + \beta U(C_2) \\ & + \lambda \left(Y_1 + \frac{Y_2}{1+r} - C_1 - \frac{C_2}{1+r} \right) \\ \rightarrow \quad & U'(C_1) = \lambda, \quad \beta U'(C_2) = \frac{\lambda}{1+r} \end{aligned}$$

Either way, end up with the same **Euler equation**:

$$\underbrace{U'(C_1)}_{\text{MU if consume one more unit this period}} = \underbrace{\beta(1+r)U'(C_2)}_{\substack{\text{MU if save one more unit this period} \\ \rightarrow \text{consume } 1+r \text{ more next period}}}$$

With the budget constraint(s), uniquely pins down C_1, C_2, S

Two examples

1. If **rate of time preference = interest rate**:
2. If **log utility**:

Two examples

1. If **rate of time preference = interest rate**: *perfect consumption smoothing*

$$U'(C_1) = \underbrace{\beta(1+r)}_{=1} U'(C_2) \rightarrow C_1 = C_2$$

2. If **log utility**:

Two examples

1. If rate of time preference = interest rate: *perfect consumption smoothing*

$$U'(C_1) = \underbrace{\beta(1+r)}_{=1} U'(C_2) \rightarrow C_1 = C_2$$

so period-1 savings are

$$S = Y_1 - C_1 = \frac{Y_1 - Y_2}{2+r}$$

2. If **log utility**:

Two examples

1. If rate of time preference = interest rate: *perfect consumption smoothing*

$$U'(C_1) = \underbrace{\beta(1+r)}_{=1} U'(C_2) \rightarrow C_1 = C_2$$

so period-1 savings are

$$S = Y_1 - C_1 = \frac{Y_1 - Y_2}{2+r}$$

2. If **log utility**:

$$\frac{1}{C_1} = \beta(1+r) \frac{1}{C_2} \rightarrow C_2 = \beta(1+r)C_1$$

Two examples

1. If rate of time preference = interest rate: *perfect consumption smoothing*

$$U'(C_1) = \underbrace{\beta(1+r)}_{=1} U'(C_2) \rightarrow C_1 = C_2$$

so period-1 savings are

$$S = Y_1 - C_1 = \frac{Y_1 - Y_2}{2 + r}$$

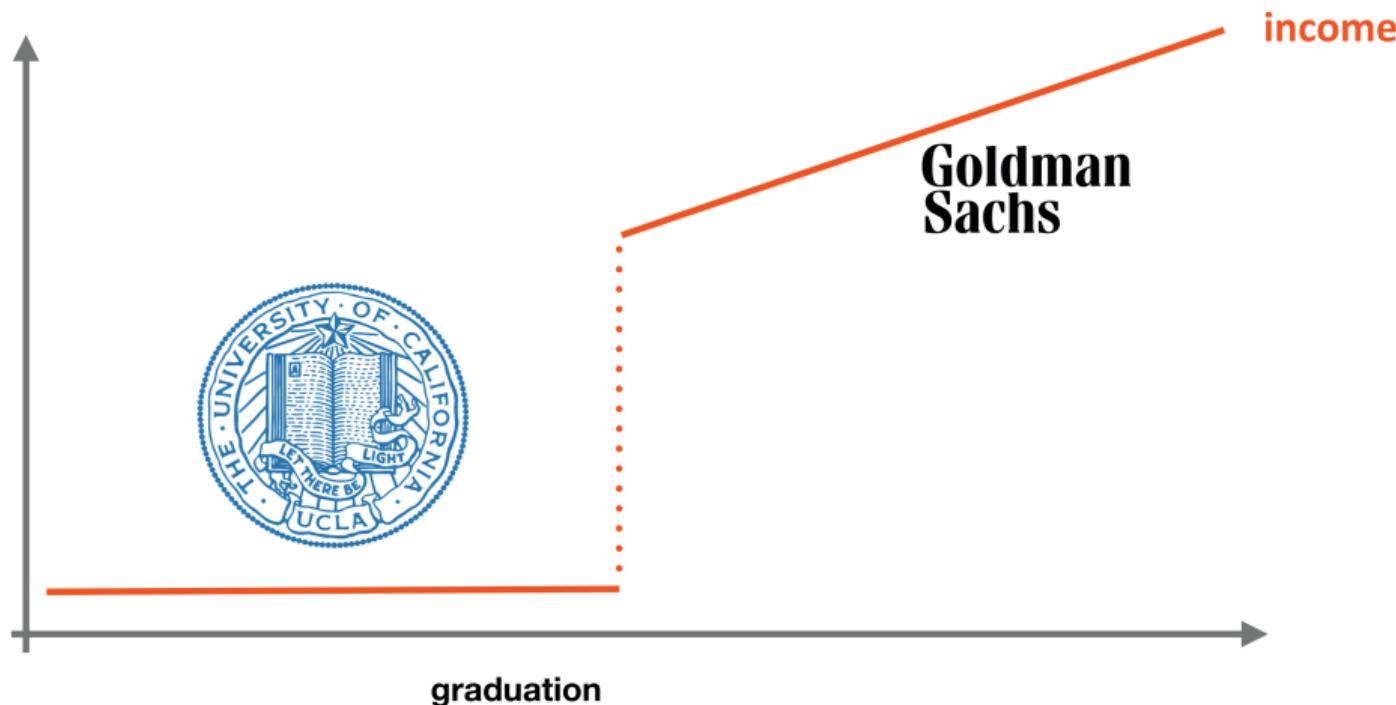
2. If **log utility**:

$$\frac{1}{C_1} = \beta(1+r) \frac{1}{C_2} \rightarrow C_2 = \beta(1+r)C_1$$

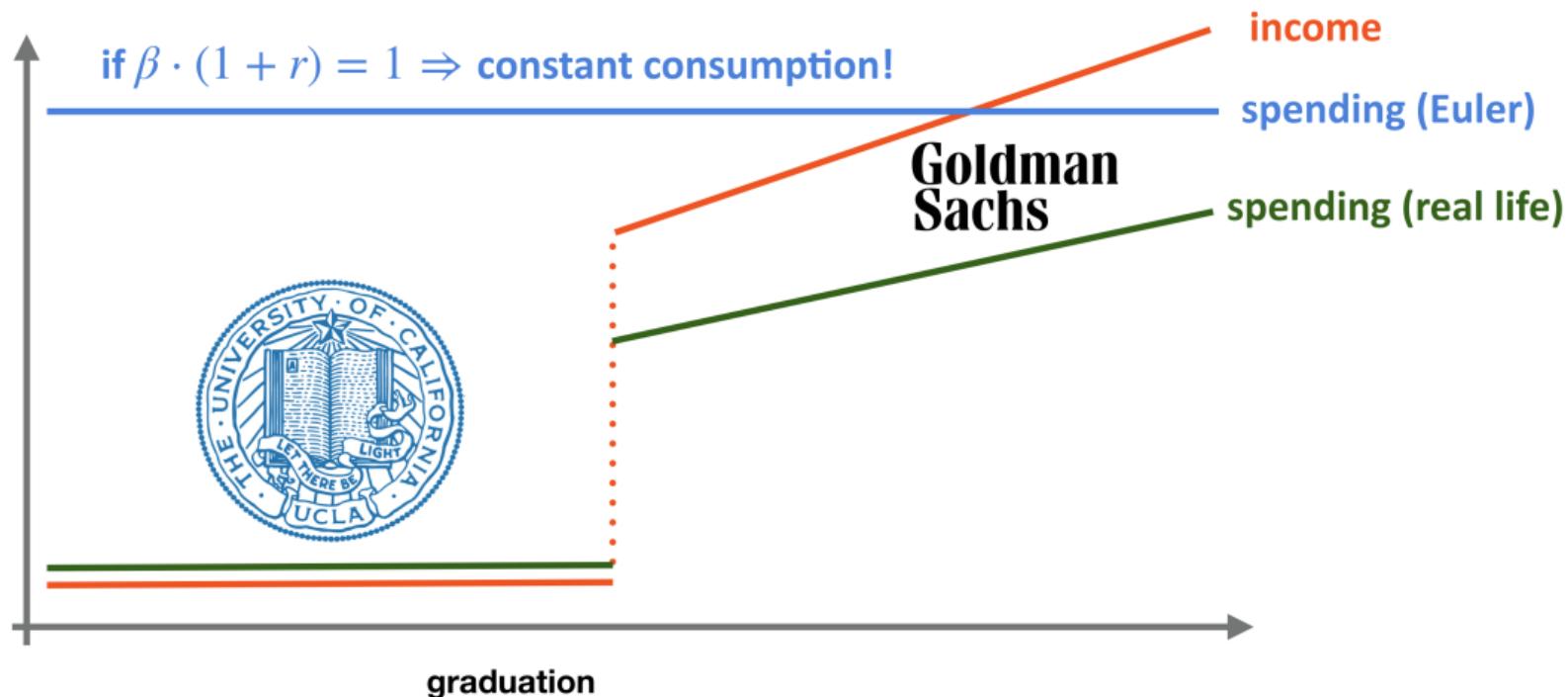
so period-1 savings are

$$S = Y_1 - C_1 = \frac{\beta Y_1 - \frac{Y_2}{1+r}}{1 + \beta}$$

Are you on your Euler equation?



Are you on your Euler equation?



Infinite-horizon consumption-saving

Infinite-horizon utility function

- Why ∞ ? Clearly people do not live forever...
 - Yes, but each generation does care about the next one → **saves to leave bequests**
 - So could interpret each person as part of an “infinite dynasty”
 - Practically, the math for infinite lives is nicer than for finite lives
- Label periods by $t = 0, 1, 2, \dots$ (start at date 0). Consumption at date $t \equiv C_t$.
- So now **lifetime utility** is

$$U(C_0) + \beta U(C_1) + \beta^2 U(C_2) + \dots = \sum_{t=0}^{\infty} \beta^t U(C_t)$$

Infinite-horizon utility function

- Why ∞ ? Clearly people do not live forever...
 - Yes, but each generation does care about the next one → **saves to leave bequests**
 - So could interpret each person as part of an “infinite dynasty”
 - Practically, the math for infinite lives is nicer than for finite lives
- Label periods by $t = 0, 1, 2, \dots$ (start at date 0). Consumption at date $t \equiv C_t$.
- So now **lifetime utility** is

$$U(C_0) + \beta U(C_1) + \beta^2 U(C_2) + \dots = \sum_{t=0}^{\infty} \beta^t U(C_t)$$

(can we do this if $\beta = 1$?)

Infinite-horizon per-period budget constraints

- Now agents can save over **many periods**
 - $S_t \equiv$ stock of savings going into period t , earns interest rate r
 - $S_0 \equiv$ *initial wealth* at the beginning of period 0
- Get an **infinite sequence of per-period (flow) budget constraints**

$$C_0 + S_1 = (1 + r)S_0 + Y_0$$

$$C_1 + S_2 = (1 + r)S_1 + Y_1$$

$$C_2 + S_3 = (1 + r)S_2 + Y_2$$

⋮

Infinite-horizon per-period budget constraints

- Now agents can save over **many periods**
 - S_t \equiv stock of savings going into period t , earns interest rate r
 - S_0 \equiv *initial wealth* at the beginning of period 0
- Get an **infinite sequence of per-period (flow) budget constraints**

$$C_0 + S_1 = (1 + r)S_0 + Y_0$$

$$C_1 + S_2 = (1 + r)S_1 + Y_1$$

$$C_2 + S_3 = (1 + r)S_2 + Y_2$$

⋮

$$C_t + S_{t+1} = (1 + r)S_t + Y_t \quad \text{for } t = 0, 1, 2, \dots$$

Infinite-horizon lifetime budget constraint

- We can again combine all the per-period constraints into a **lifetime constraint**
- Solve out per-period consumption C_t and take **PDVs** from date 0

$$C_0 = (1 + r)S_0 + Y_0 - S_1 \quad \rightarrow$$

$$C_0 = (1 + r)S_0 + Y_0 - S_1$$

$$C_1 = (1 + r)S_1 + Y_1 - S_2 \quad \rightarrow$$

$$\frac{C_1}{1 + r} = S_1 + \frac{Y_1}{1 + r} - \frac{S_2}{1 + r}$$

$$C_2 = (1 + r)S_2 + Y_2 - S_3 \quad \rightarrow$$

$$\frac{C_2}{(1 + r)^2} = \frac{S_2}{1 + r} + \frac{Y_2}{(1 + r)^2} - \frac{S_3}{(1 + r)^2}$$

⋮

⋮

Infinite-horizon lifetime budget constraint

- We can again combine all the per-period constraints into a **lifetime constraint**
- Solve out per-period consumption C_t and take **PDVs** from date 0

$$C_0 = (1 + r)S_0 + Y_0 - S_1 \quad \rightarrow$$

$$C_0 = (1 + r)S_0 + Y_0 - \cancel{S_1}$$

$$C_1 = (1 + r)S_1 + Y_1 - S_2 \quad \rightarrow$$

$$\frac{C_1}{1 + r} = \cancel{S_1} + \frac{Y_1}{1 + r} - \cancel{\frac{S_2}{1 + r}}$$

$$C_2 = (1 + r)S_2 + Y_2 - S_3 \quad \rightarrow$$

$$\frac{C_2}{(1 + r)^2} = \cancel{\frac{S_2}{1 + r}} + \frac{Y_2}{(1 + r)^2} - \frac{S_3}{(1 + r)^2}$$

⋮

⋮

Infinite-horizon lifetime budget constraint

- We can again combine all the per-period constraints into a **lifetime constraint**
- Solve out per-period consumption C_t and take **PDVs** from date 0

$$C_0 = (1 + r)S_0 + Y_0 - S_1 \quad \rightarrow \quad C_0 = (1 + r)S_0 + Y_0 - \cancel{S_1}$$

$$C_1 = (1 + r)S_1 + Y_1 - S_2 \quad \rightarrow \quad \frac{C_1}{1 + r} = \cancel{S_1} + \frac{Y_1}{1 + r} - \cancel{\frac{S_2}{1 + r}}$$

$$C_2 = (1 + r)S_2 + Y_2 - S_3 \quad \rightarrow \quad \frac{C_2}{(1 + r)^2} = \cancel{\frac{S_2}{1 + r}} + \frac{Y_2}{(1 + r)^2} - \frac{S_3}{(1 + r)^2}$$

⋮

⋮

$$\sum_{t=0}^{\infty} \frac{C_t}{(1 + r)^t} = (1 + r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1 + r)^t}$$

Two formulations of the infinite-horizon consumption-saving problem

1. The **per-period** formulation:

$$\max_{\{C_t, S_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.} \quad C_t + S_{t+1} = (1+r)S_t + Y_t \quad \text{for } t = 0, 1, 2, \dots$$

2. The **lifetime** formulation:

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.} \quad \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} = (1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t}$$

Two formulations of the infinite-horizon consumption-saving problem

1. The **per-period** formulation: solve via **substitution**

$$\max_{\{C_t, S_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.} \quad C_t + S_{t+1} = (1+r)S_t + Y_t \quad \text{for } t = 0, 1, 2, \dots$$

2. The **lifetime** formulation: solve via **Lagrangian**

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.} \quad \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} = (1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t}$$

*continuous time: solve via **Hamiltonian***
(Barro and Sala-i Martin, 2004)

The Lagrangian for the lifetime formulation is

$$\mathcal{L}(\{C_t\}) = \sum_{t=0}^{\infty} \beta^t U(C_t) + \lambda \left[(1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t} - \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} \right]$$

The Lagrangian for the lifetime formulation is

$$\mathcal{L}(\{C_t\}) = \sum_{t=0}^{\infty} \beta^t U(C_t) + \lambda \left[(1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t} - \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} \right]$$

The first-order conditions w.r.t. any particular C_t and C_{t+1} are

$$0 = \frac{\partial \mathcal{L}}{\partial C_t} = \beta^t U'(C_t) - \lambda \frac{1}{(1+r)^t}$$

$$0 = \frac{\partial \mathcal{L}}{\partial C_{t+1}} = \beta^{t+1} U'(C_{t+1}) - \lambda \frac{1}{(1+r)^{t+1}}$$

The Lagrangian for the lifetime formulation is

$$\mathcal{L}(\{C_t\}) = \sum_{t=0}^{\infty} \beta^t U(C_t) + \lambda \left[(1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t} - \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} \right]$$

The first-order conditions w.r.t. any particular C_t and C_{t+1} are

$$0 = \frac{\partial \mathcal{L}}{\partial C_t} = \beta^t U'(C_t) - \lambda \frac{1}{(1+r)^t}$$

$$0 = \frac{\partial \mathcal{L}}{\partial C_{t+1}} = \beta^{t+1} U'(C_{t+1}) - \lambda \frac{1}{(1+r)^{t+1}}$$

Solving for λ and rearranging recovers the **Euler equation**: for $t = 0, 1, 2, \dots$

$$U'(C_t) = \beta(1+r)U'(C_{t+1})$$

Characterizing the solution

The **solution** is then the unique set of C_0, C_1, C_2, \dots that solve

$$U'(C_t) = \beta(1+r)U'(C_{t+1}) \quad \forall t, \quad \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} = (1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t}$$

Characterizing the solution

The **solution** is then the unique set of C_0, C_1, C_2, \dots that solve

$$U'(C_t) = \beta(1+r)U'(C_{t+1}) \quad \forall t, \quad \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} = (1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t}$$

With **log utility**, the Euler equation simplifies to $C_{t+1} = \beta(1+r)C_t$, so

$$C_t = \beta^t(1+r)^t C_0$$

$$\begin{aligned} C_0 &= \left((1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t} \right) / \left(\sum_{t=0}^{\infty} \beta^t \right) \\ &= \left((1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t} \right) / \left(\frac{1}{1-\beta} \right) \\ &= (1-\beta) \left((1+r)S_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t} \right) \end{aligned}$$

Taking stock

- What else should C_t and I_t depend on other than **current income Y_t** ?

Taking stock

- What else should C_t and I_t depend on other than **current income Y_t** ?
- Take *microfounded* approach:
saving is **chosen** by a rational utility-maximizing rep. HH over ∞ periods

Taking stock

- What else should C_t and I_t depend on other than **current income Y_t** ?
- Take *microfounded* approach:
saving is **chosen** by a rational utility-maximizing rep. HH over ∞ periods

future Y_t ? age/life cycle?

past Y_t ? wealth?

impatience?

interest rates? taxes?

uncertainty?

borrowing constraints?

Taking stock

- What else should C_t and I_t depend on other than **current income Y_t** ?
- Take *microfounded* approach:
 - saving is **chosen** by a rational utility-maximizing rep. HH over ∞ periods
 - future Y_t ? age/life cycle?
 - past Y_t ? wealth? So, turning back to **growth...**
 - impatience?
 - interest rates? taxes? **opportunities** vs. **necessities**
 - uncertainty?
 - borrowing constraints? (modern macro inc. all these *and more!*)

Revisiting the Solow model

Consumption in the Solow model, revisited

Recall from last week...

$$y_t^{\text{BGP}} = A_t \left(\tilde{k}^{\text{ss}} \right)^\alpha = A_t \left(\frac{s}{\delta + g_A + g_L} \right)^{\frac{\alpha}{1-\alpha}}$$

... so should we just max out w/ $s = 1$?

Consumption in the Solow model, revisited

Recall from last week...

$$y_t^{\text{BGP}} = A_t \left(\tilde{k}^{\text{ss}} \right)^\alpha = A_t \left(\frac{s}{\delta + g_A + g_L} \right)^{\frac{\alpha}{1-\alpha}}$$

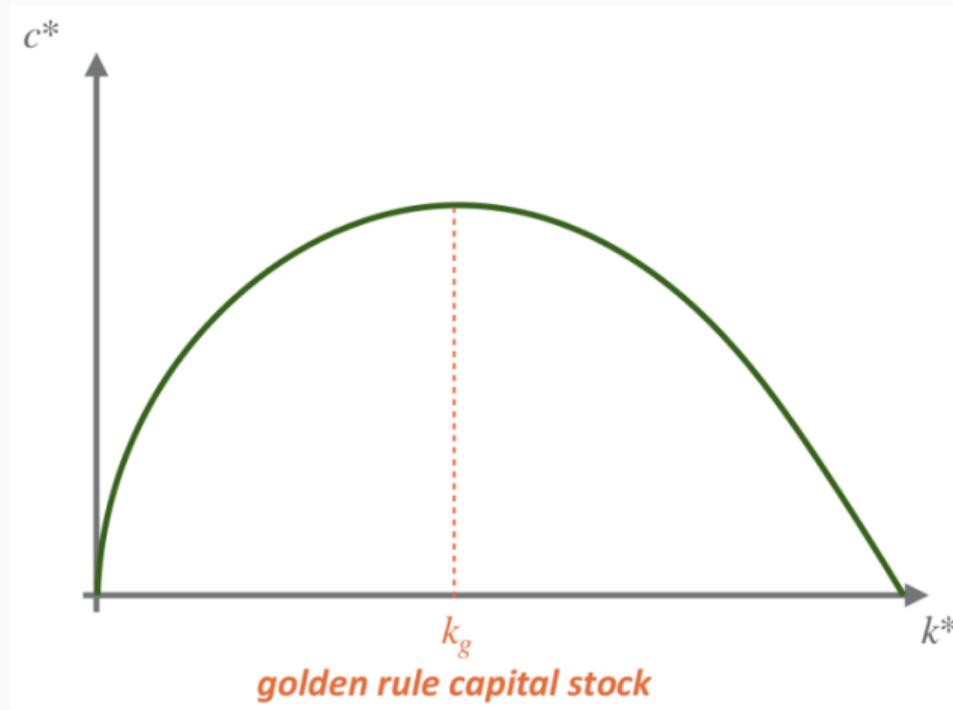
... so should we just max out w/ $s = 1$?

Not if what matters for utility is **consumption**:

$$c_t^{\text{BGP}} = (1 - s) y_t^{\text{BGP}} = (1 - s) A_t \left(\frac{s}{\delta + g_A + g_L} \right)^{\frac{\alpha}{1-\alpha}}$$

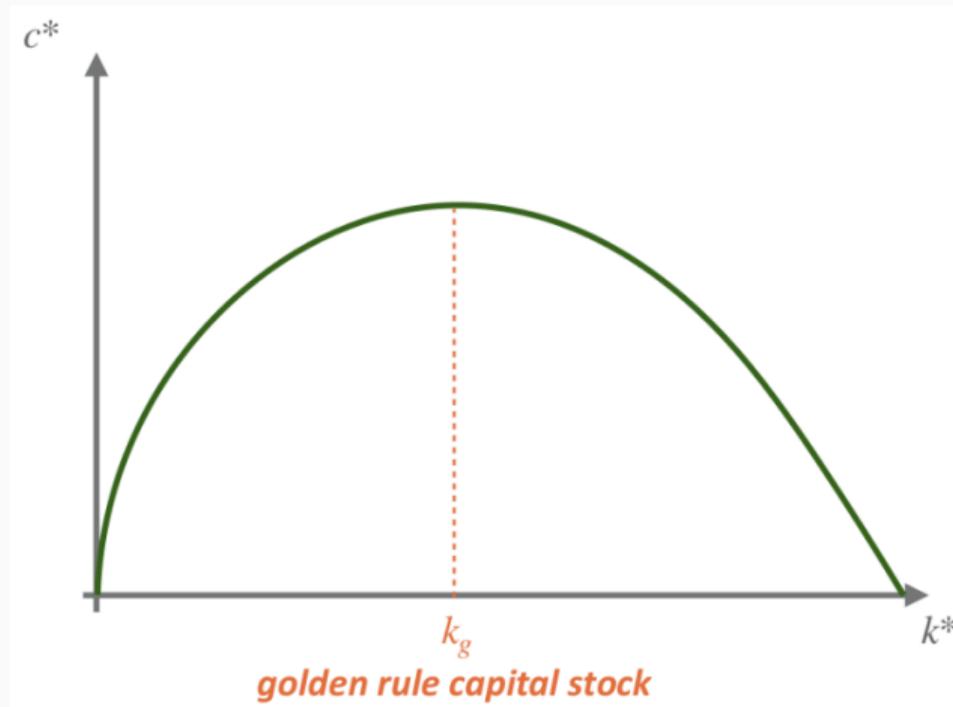
The “golden rule” to optimize steady-state consumption

(Phelps, 1961)



The “golden rule” to optimize steady-state consumption

(Phelps, 1961)

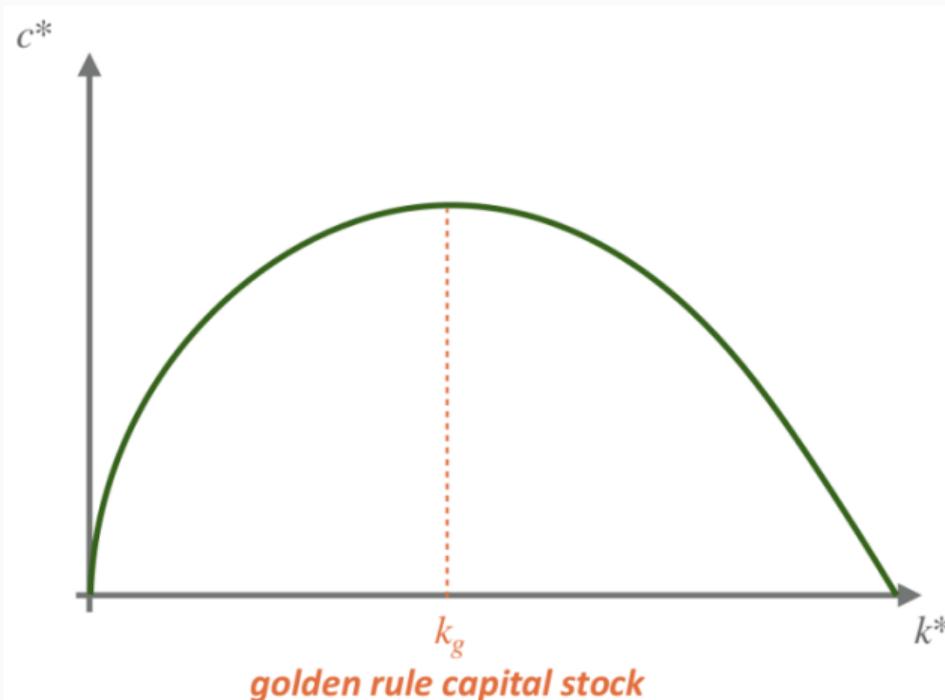


Let's work w/ **detrended** consumption:

$$\tilde{c}^{ss} \equiv \frac{c_t^{\text{BGP}}}{A_t} = (1-s) \frac{y_t^{\text{BGP}}}{A_t} \equiv (1-s)\tilde{y}^{ss}$$

The “golden rule” to optimize steady-state consumption

(Phelps, 1961)



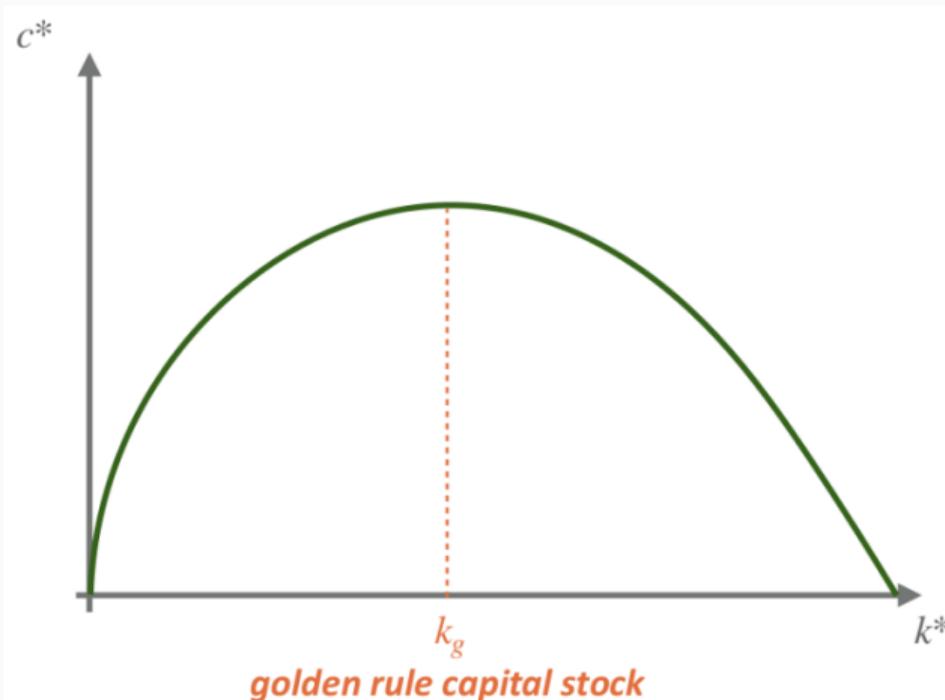
Let's work w/ **detrended** consumption:

$$\tilde{c}^{ss} \equiv \frac{c_t^{\text{BGP}}}{A_t} = (1-s) \frac{y_t^{\text{BGP}}}{A_t} \equiv (1-s)\tilde{y}^{ss}$$

← *What is that curve?*

The “golden rule” to optimize steady-state consumption

(Phelps, 1961)



Let's work w/ **detrended** consumption:

$$\tilde{c}^{ss} \equiv \frac{c_t^{\text{BGP}}}{A_t} = (1-s) \frac{y_t^{\text{BGP}}}{A_t} \equiv (1-s)\tilde{y}^{ss}$$

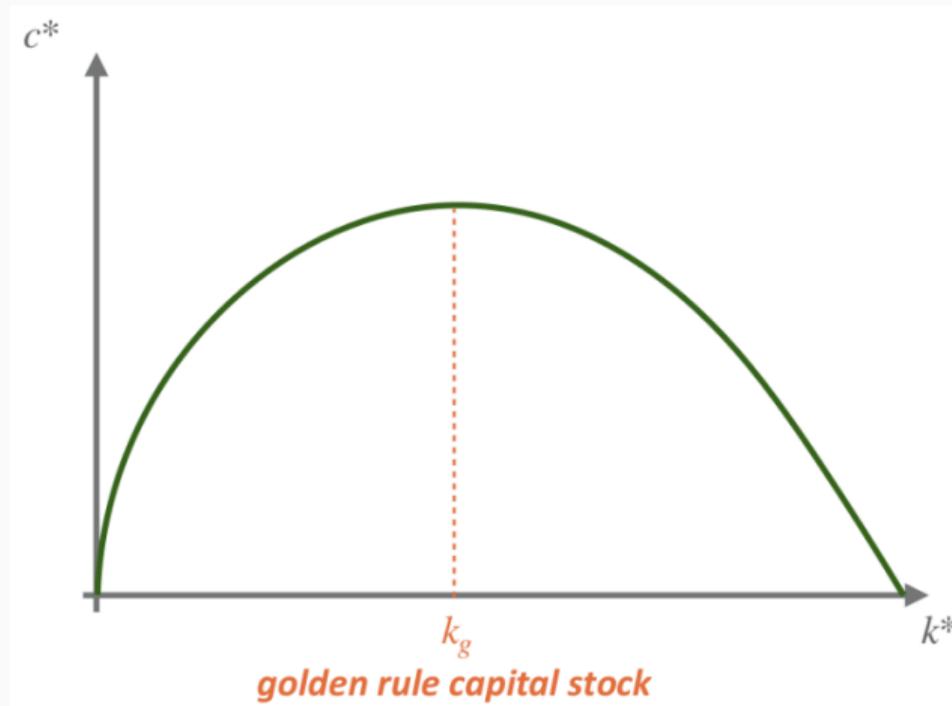
← **What is that curve?**

$$\tilde{c}^{ss} = (1 - \textcolor{brown}{s}) \cdot \left[\tilde{k}^{ss}(\textcolor{brown}{s}) \right]^\alpha$$

All values of \tilde{c} consistent with $g_{\tilde{k}} = 0$

The “golden rule” to optimize steady-state consumption

(Phelps, 1961)



Solving $\max_s \tilde{c}^{ss}$:

$$\tilde{y}^{ss} = (1 - s) \frac{\partial \tilde{y}^{ss}}{\partial s}$$

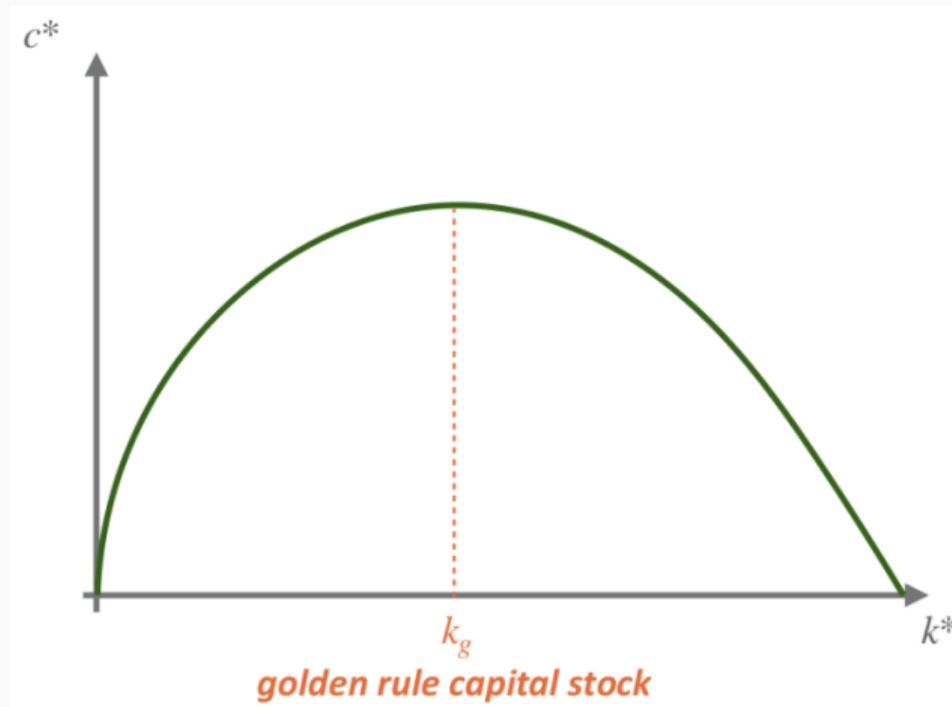
$$\frac{s}{1 - s} = \frac{\partial \tilde{y}^{ss}}{\partial s} \frac{s}{\tilde{y}^{ss}}$$

$$\frac{s}{1 - s} = \frac{\alpha}{1 - \alpha}$$

$$s^{\text{GR}} = \alpha$$

The “golden rule” to optimize steady-state consumption

(Phelps, 1961)



Solving $\max_s \tilde{c}^{ss}$:

$$\tilde{y}^{ss} = (1 - s) \frac{\partial \tilde{y}^{ss}}{\partial s}$$

$$\frac{s}{1 - s} = \frac{\partial \tilde{y}^{ss}}{\partial s} \frac{s}{\tilde{y}^{ss}}$$

$$\frac{s}{1 - s} = \frac{\alpha}{1 - \alpha}$$

$$s^{\text{GR}} = \alpha$$

What's the intuition?

The Solow model in discrete time

As we switch into discrete time, let's simplify: $L_t = 1, A_t = 1 \forall t \rightarrow K_t = k_t = \tilde{k}_t$

The Solow model in discrete time

As we switch into discrete time, let's simplify: $L_t = 1, A_t = 1 \forall t \rightarrow K_t = k_t = \tilde{k}_t$

1. aggregate production function:

$$y_t = F(k_t, 1) = k_t^\alpha$$

The Solow model in discrete time

As we switch into discrete time, let's simplify: $L_t = 1$, $A_t = 1 \ \forall t \rightarrow K_t = k_t = \tilde{k}_t$

1. aggregate production function:

$$y_t = F(k_t, 1) = k_t^\alpha$$

2. capital accumulation:

$$\frac{k_{t+1} - k_t}{\Delta t} = sF(k_t, 1) - \delta k_t$$

The Solow model in discrete time

As we switch into discrete time, let's simplify: $L_t = 1$, $A_t = 1 \ \forall t \rightarrow K_t = k_t = \tilde{k}_t$

1. aggregate production function:

$$y_t = F(k_t, 1) = k_t^\alpha$$

2. capital accumulation:

$$k_{t+1} = sF(k_t, 1) + (1 - \delta)k_t$$

The Solow model in discrete time

As we switch into discrete time, let's simplify: $L_t = 1, A_t = 1 \forall t \rightarrow \mathbf{K}_t = k_t = \tilde{k}_t$

1. aggregate production function:

$$y_t = F(k_t, 1) = k_t^\alpha$$

2. capital accumulation:

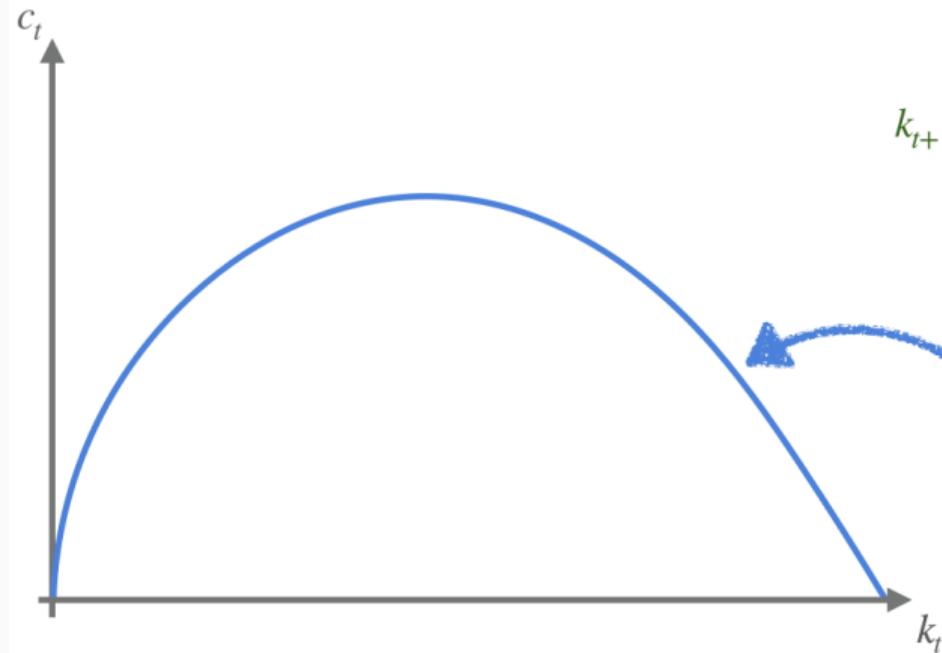
$$k_{t+1} = sF(k_t, 1) + (1 - \delta)k_t$$

but we can split that into...

$$k_{t+1} = [\mathbf{F}(k_t, 1) - c_t] + (1 - \delta)k_t, \quad c_t = (1 - s)F(k_t, 1)$$

A steady state is a point where $\frac{k_{t+1} - k_t}{\Delta t} = 0 \rightarrow k_{t+1} = k_t \rightarrow c_{t+1} = c_t$

(Re)introducing the c - k phase diagram

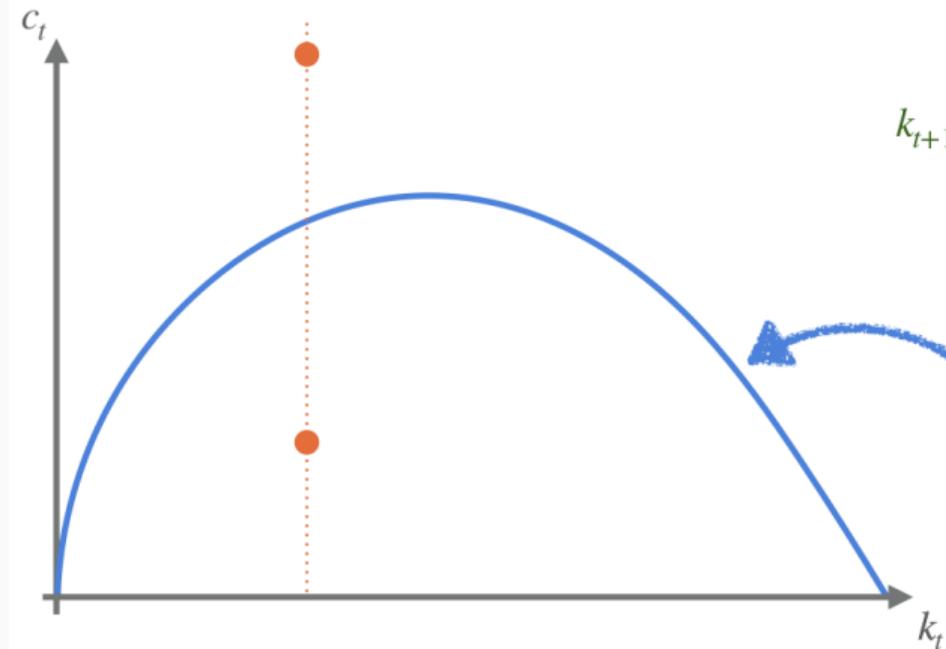


the law of motion:

$$k_{t+1} = F(k_t, 1) - c_t + (1 - \delta) k_t$$

Q: How can we find the points for which $k_{t+1} = k_t$?

(Re)introducing the c - k phase diagram



the law of motion:

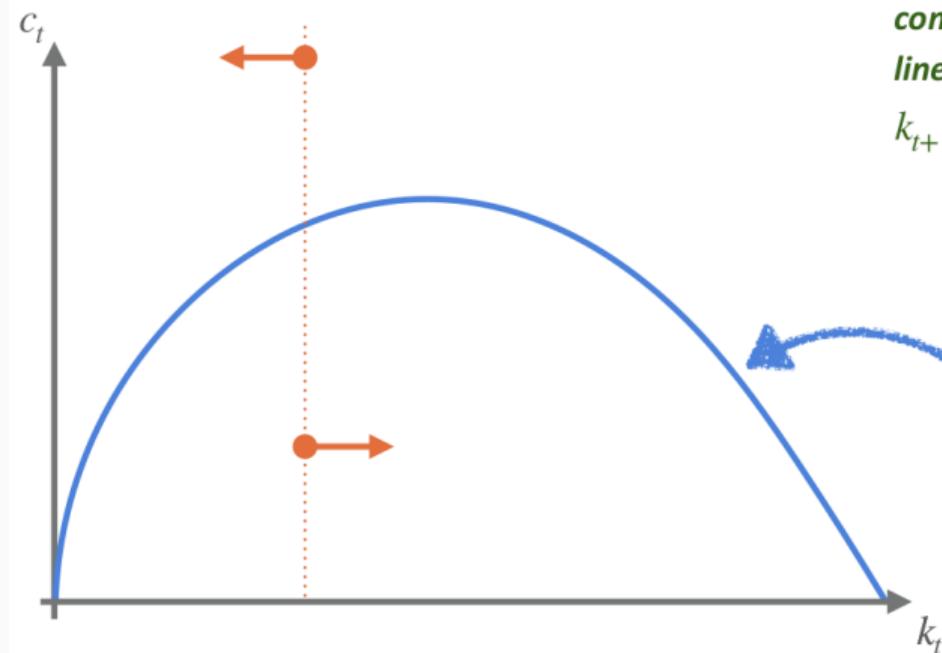
$$k_{t+1} = F(k_t, 1) - c_t + (1 - \delta) k_t$$

Q: How can we find the points for which $k_{t+1} = k_t$?

$$\rightarrow c_t = F(k_t, 1) - \delta k_t \quad \text{We also call this a locus}$$

Q: What happens if, for a given k_t we choose consumption above or below the blue line?

(Re)introducing the c - k phase diagram



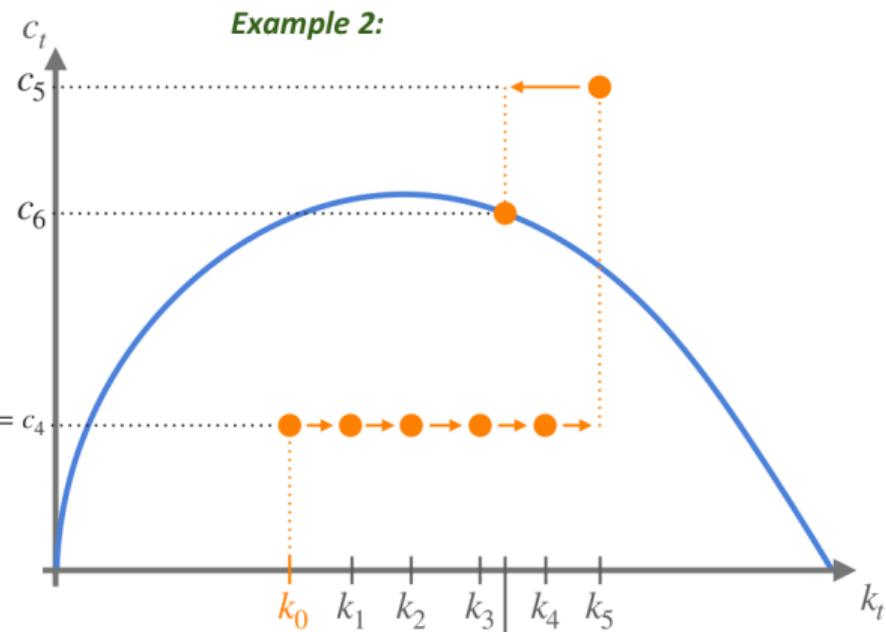
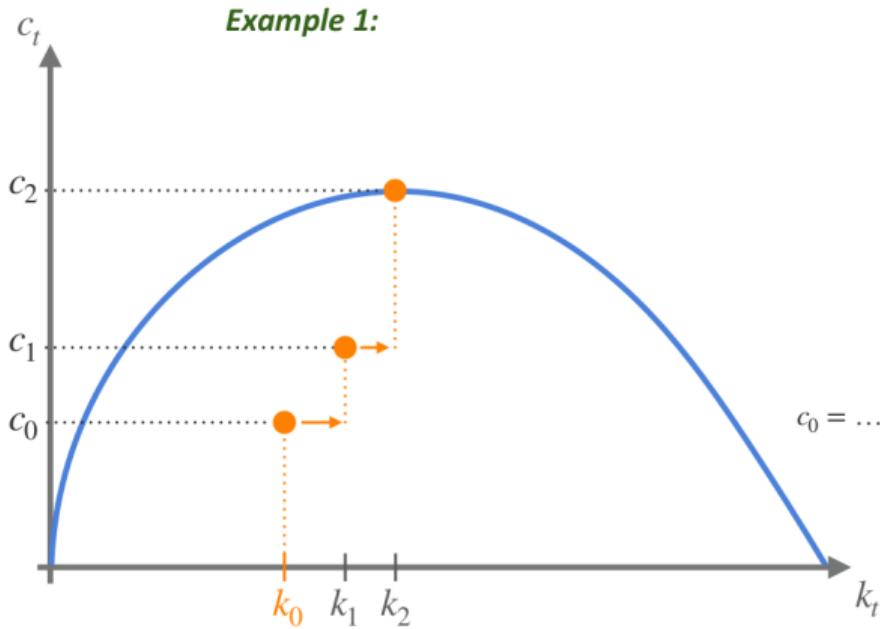
For each point (k_t, c_t) in the diagram, we can compute next period's capital stock k_{t+1} , in line with the law of motion:

$$k_{t+1} = F(k_t, 1) - c_t + (1 - \delta) k_t$$

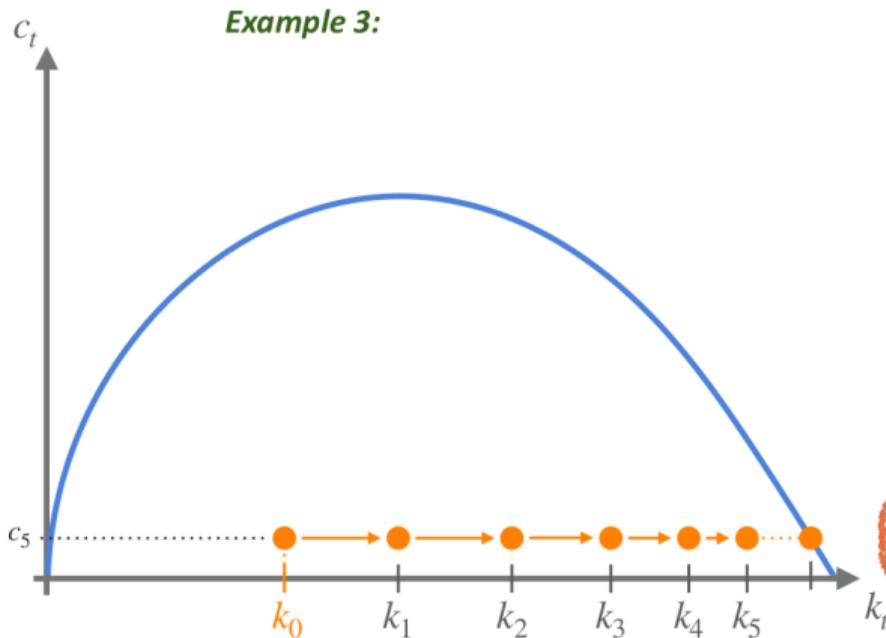
Q: How can we find the points for which $k_{t+1} = k_t$?
→ $c_t = F(k_t, 1) - \delta k_t$ We also call this a locus

Q: What happens if, for a given k_t we choose consumption above or below the blue line?

For any k_0 , different consumption rules \rightarrow different transition paths

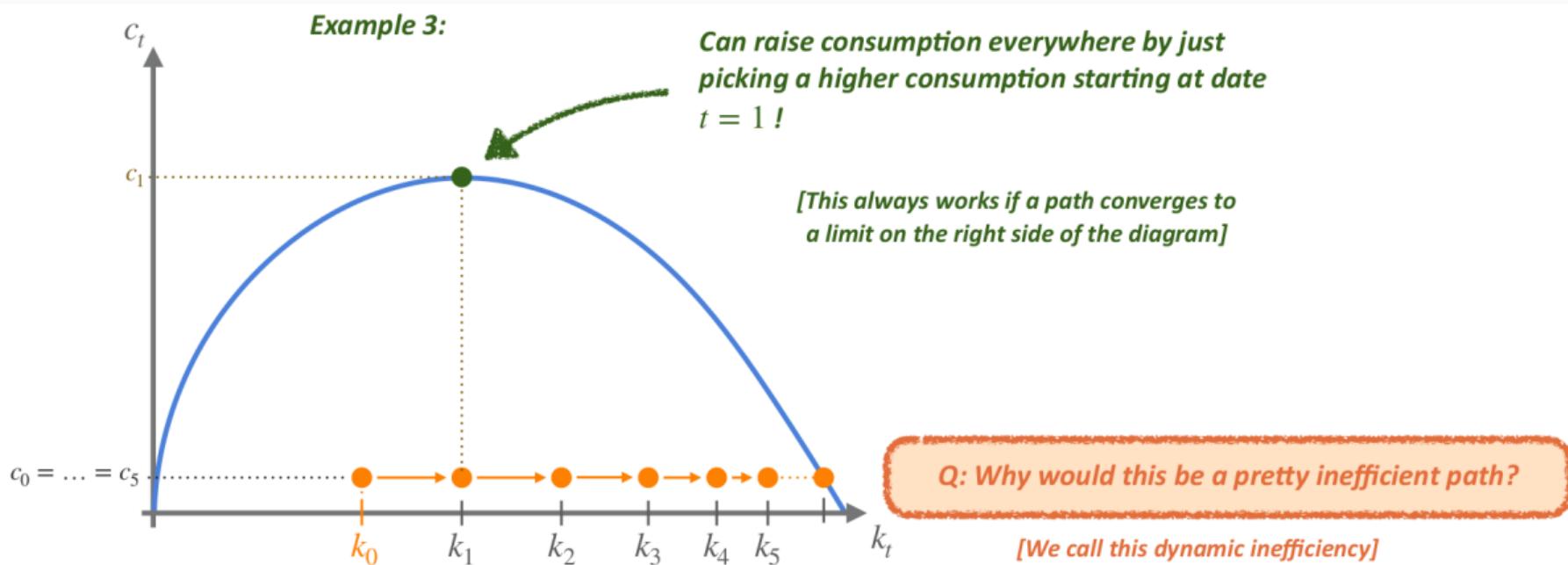


Some paths are clearly inferior to others...

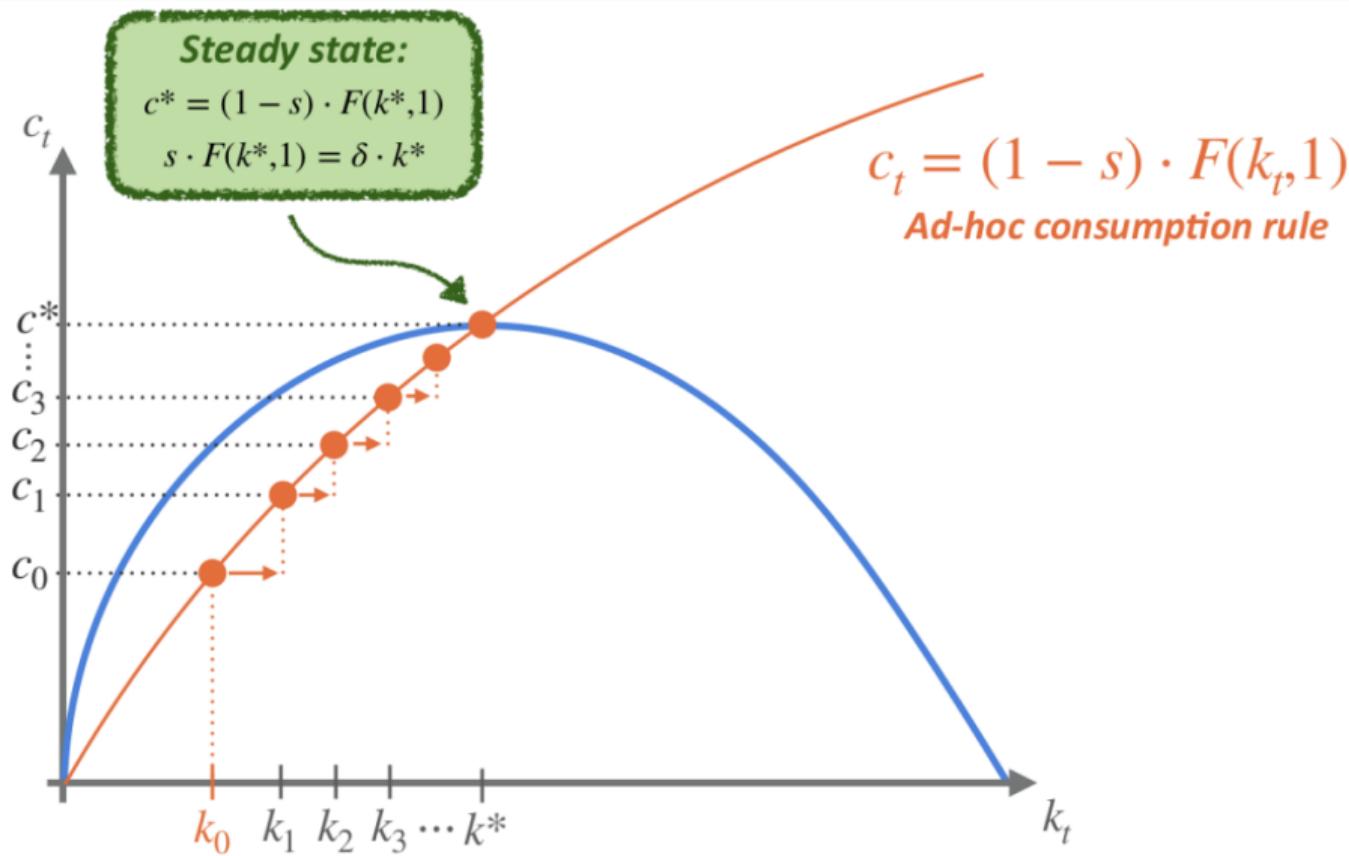


Q: Why would this be a pretty inefficient path?

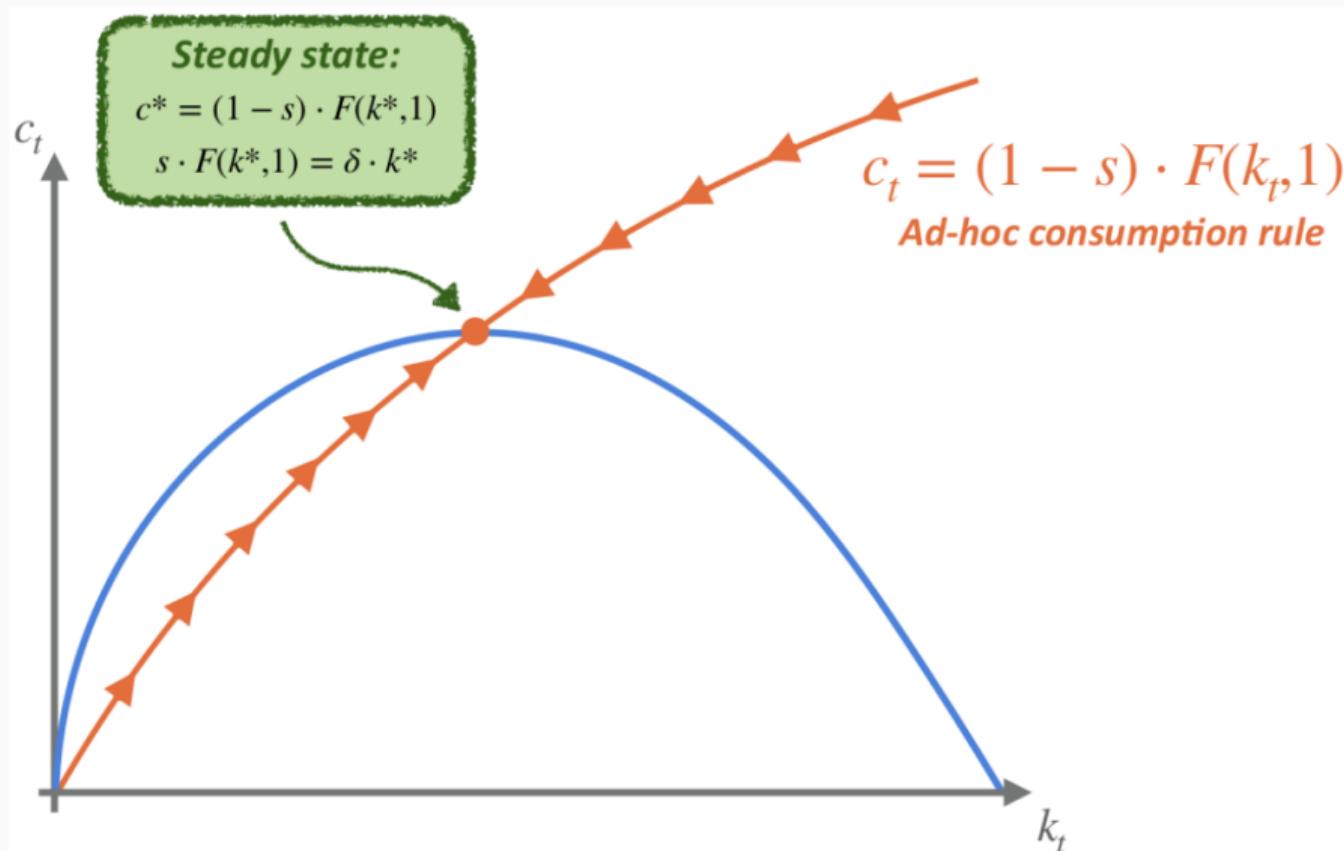
Some paths are clearly inferior to others...



Solow consumption rule \rightarrow just one particular path



Solow consumption rule \rightarrow just one particular path

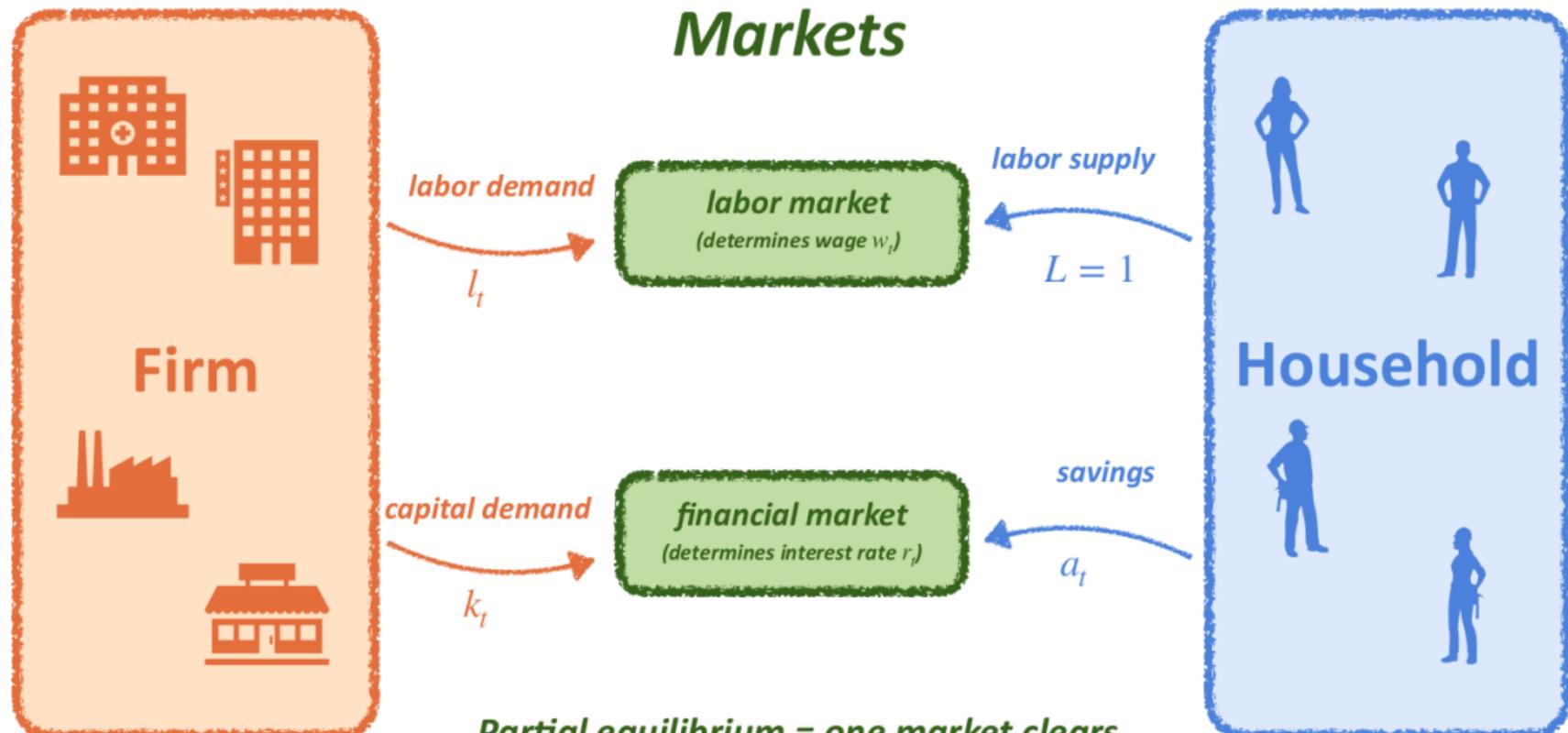


The Ramsey-Cass-Koopmans model

Overview

- The **Ramsey-Cass-Koopmans** model will be our first **fully modern** macro model
- **Main idea:** Want to replace the ad-hoc consumption rule of the Solow model...
 - ...with consumption behavior from a utility-maximizing ∞ -horizon household
- Two approaches to set up this model
 - Describe as **general equilibrium**
 - Representative household & firm, interact thru markets *[will focus on this one!]*
 - Describe as **planning problem**
 - A “benevolent planner” maximizes household utility s.t. resource constraints
- **Note:** Both yield the same, so we say that the model is **first-best efficient**

General equilibrium



General equilibrium = all markets clear simultaneously

Household optimization problem

- Household earns **wage** w_t and accumulates a stock of **assets** a_t entering period t
 - *last class*: income was denoted by Y_t and savings by S_t
 - in general equilibrium, using w_t and a_t is somewhat more common

Household optimization problem

- Household earns **wage** w_t and accumulates a stock of **assets** a_t entering period t
 - *last class*: income was denoted by Y_t and savings by S_t
 - in general equilibrium, using w_t and a_t is somewhat more common
- Household now faces **time-varying** interest rate r_t in period t

Household optimization problem

- Household earns **wage** w_t and accumulates a stock of **assets** a_t entering period t
 - *last class*: income was denoted by Y_t and savings by S_t
 - in general equilibrium, using w_t and a_t is somewhat more common
- Household now faces **time-varying** interest rate r_t in period t
- Household then solves for its **optimal consumption path** $\{c_t\}$:

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t) \quad \text{s.t.} \quad c_t + a_{t+1} = (1 + r_t)a_t + w_t \quad \text{for } t = 0, 1, 2, \dots$$

Household optimization problem

- Household earns **wage** w_t and accumulates a stock of **assets** a_t entering period t
 - *last class*: income was denoted by Y_t and savings by S_t
 - in general equilibrium, using w_t and a_t is somewhat more common
- Household now faces **time-varying** interest rate r_t in period t
- Household then solves for its **optimal consumption path** $\{c_t\}$:

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t) \quad \text{s.t.} \quad c_t + a_{t+1} = (1 + r_t)a_t + w_t \quad \text{for } t = 0, 1, 2, \dots$$

which is characterized by the **Euler equation**

$$U'(c_t) = \beta(1 + r_{t+1})U'(c_{t+1}) \quad \text{for } t = 0, 1, 2, \dots$$

and **budget feasibility**

(recall from last class!)

Firm optimization problem

- In a general equilibrium model, we also need to describe:
 - **what** the household saves in...
 - ... and **who** pays the wage
- Suppose there is a representative firm that, **each period**:
 - Rents capital k_t from the household at rate $R_t = r_t + \delta$
 - compensate household for *opportunity cost* (r_t) and *depreciation* (δ)
 - *note*: what we called r in the Solow model would here be R_t
 - Employs the household at wage w_t

Firm optimization problem

- In a general equilibrium model, we also need to describe:
 - **what** the household saves in...
 - ... and **who** pays the wage
- Suppose there is a representative firm that, **each period**:
 - Rents capital k_t from the household at rate $R_t = r_t + \delta$
 - compensate household for *opportunity cost* (r_t) and *depreciation* (δ)
 - *note*: what we called r in the Solow model would here be R_t
 - Employs the household at wage w_t

$$\max_{k_t, \ell_t} \underbrace{F(k_t, \ell_t)}_{\text{revenue}} - \underbrace{(r_t + \delta)k_t}_{\substack{\text{capital} \\ \text{expenditures}}} - \underbrace{w_t \ell_t}_{\text{wage} \atop \text{payments}}$$

Firm optimization problem

- In a general equilibrium model, we also need to describe:
 - **what** the household saves in...
 - ... and **who** pays the wage
- Suppose there is a representative firm that, **each period**:
 - Rents capital k_t from the household at rate $R_t = r_t + \delta$
 - compensate household for *opportunity cost* (r_t) and *depreciation* (δ)
 - *note*: what we called r in the Solow model would here be R_t
 - Employs the household at wage w_t

$$\max_{k_t, \ell_t} \underbrace{F(k_t, \ell_t)}_{\text{revenue}} - \underbrace{(r_t + \delta)k_t}_{\text{capital expenditures}} - \underbrace{w_t \ell_t}_{\text{wage payments}}$$

$$\frac{\partial F(k_t, \ell_t)}{\partial k_t} \equiv \mathbf{F}_k(k_t, \ell_t) = r_t + \delta$$
$$\frac{\partial F(k_t, \ell_t)}{\partial \ell_t} \equiv \mathbf{F}_\ell(k_t, \ell_t) = w_t$$

Defining equilibrium

A **competitive equilibrium** consists of quantities $\{c_t, a_t, k_t, \ell_t\}$ and prices $\{r_t, w_t\}$ s.t.

1. The household optimally picks $\{c_t, a_t\}$ given $\{r_t, w_t\}$

$$U'(c_t) = \beta(1 + r_{t+1})U'(c_{t+1}), \quad c_t + a_{t+1} = (1 + r_t)a_t + w_t \quad \text{for } t = 0, 1, 2, \dots$$

2. The firm optimally picks $\{k_t, \ell_t\}$ given $\{r_t, w_t\}$

$$F_k(k_t, \ell_t) = r_t + \delta, \quad F_\ell(k_t, \ell_t) = w_t$$

3. The asset market clears: $k_t = a_t$

4. The labor market clears: $\ell_t = 1$

5. The resource constraint holds:

(check: already implied by #1-4)

$$k_{t+1} = F(k_t, 1) - c_t + (1 - \delta)k_t$$

Ramsey-Cass-Koopmans vs. Solow

Both have **same** resource constraint... ...but **different** consumption rules:

$$k_{t+1} = F(k_t, 1) - c_t + (1 - \delta)k_t$$

$$c_t = (1 - s)F(k_t, 1)$$

vs.

$$U'(c_t) = \beta[1 + r_{t+1}]U'(c_{t+1})$$

Ramsey-Cass-Koopmans vs. Solow

Both have **same** resource constraint... ...but **different** consumption rules:

$$k_{t+1} = F(k_t, 1) - c_t + (1 - \delta)k_t$$

$$c_t = (1 - s)F(k_t, 1)$$

vs.

$$U'(c_t) = \beta[1 + F_k(k_{t+1}, 1) - \delta]U'(c_{t+1})$$

Ramsey-Cass-Koopmans vs. Solow

Both have **same** resource constraint... ...but **different** consumption rules:

$$k_{t+1} = F(k_t, 1) - c_t + (1 - \delta)k_t$$

$$c_t = (1 - s)F(k_t, 1)$$

vs.

$$U'(c_t) = \beta[1 + F_k(k_{t+1}, 1) - \delta]U'(c_{t+1})$$

How can we **solve** this system? **Just like Solow!**

1. Characterize the **steady state**
2. Analyze transitions using ***c-k* phase diagram**

Steady state

A **steady state** is a pair $\{c^{ss}, k^{ss}\}$ at which **optimally**...

$$c_t = c_{t+1} = c^{ss} \quad \rightarrow \quad U'(c^{ss}) = \beta [1 + F_k(k^{ss}, 1) - \delta] U'(c^{ss})$$

$$F_k(k^{ss}, 1) = \frac{1}{\beta} - 1 + \delta$$

$$k_t = k_{t+1} = k^{ss} \quad \rightarrow \quad k^{ss} = F(k^{ss}, 1) - c^{ss} + (1 - \delta)k^{ss}$$

$$c^{ss} = F(k^{ss}, 1) - \delta k^{ss}$$

Can be shown: optimal $\{c_t, k_t\}$ always converges to $\{c^{ss}, k^{ss}\}$

(Check: What is the relationship between β and r^{ss} ?)

The “*modified* golden rule” to optimize the consumption path

Recall the **golden rule** ($s^{\text{GR}} = \alpha$):

(Cobb-Douglas, $L_t = 1$, $A_t = 1$)

$$k_{\text{Solow}}^{\text{ss}} = \left(\frac{s}{\delta + g_A + g_L} \right)^{\frac{1}{1-\alpha}} \rightarrow k^{\text{GR}} = \left(\frac{\alpha}{\delta} \right)^{\frac{1}{1-\alpha}}$$

But, from the Kaldor facts: $s \approx \frac{1}{4} < \frac{1}{3} \approx \alpha \rightarrow \text{should we be saving more?}$

The “*modified* golden rule” to optimize the consumption path

Recall the **golden rule** ($s^{\text{GR}} = \alpha$):

(Cobb-Douglas, $L_t = 1$, $A_t = 1$)

$$k_{\text{Solow}}^{\text{ss}} = \left(\frac{s}{\delta + g_A + g_L} \right)^{\frac{1}{1-\alpha}} \rightarrow k^{\text{GR}} = \left(\frac{\alpha}{\delta} \right)^{\frac{1}{1-\alpha}}$$

But, from the Kaldor facts: $s \approx \frac{1}{4} < \frac{1}{3} \approx \alpha \rightarrow$ should we be saving more?

The **modified golden rule** says *not necessarily*:

$$F_k(k^{\text{ss}}, 1) = \frac{1}{\beta} - 1 + \delta \rightarrow k^{\text{MGR}} = \left(\frac{\alpha}{\frac{1}{\beta} - 1 + \delta} \right)^{\frac{1}{1-\alpha}} < k^{\text{GR}}$$

The golden rule didn't account for impatience!

What happens away from steady state?

Substitute the **resource constraint** into the **Euler equation**

$$U'(c_t) = \beta \left[1 + F_k \underbrace{(F(k_t, 1) - c_t + (1 - \delta)k_t, 1) - \delta}_{k_{t+1}: \text{resource constraint}} \right] U'(c_{t+1})$$

What happens away from steady state?

Substitute the **resource constraint** into the **Euler equation**

$$U'(c_t) = \beta \left[1 + F_k \underbrace{(F(k_t, 1) - c_t + (1 - \delta)k_t, 1) - \delta}_{k_{t+1}: \text{resource constraint}} \right] U'(c_{t+1})$$

Cannot explicitly link consumption across periods anymore—depends on k_t (thru r_t)!

What happens away from steady state?

Substitute the **resource constraint** into the **Euler equation**

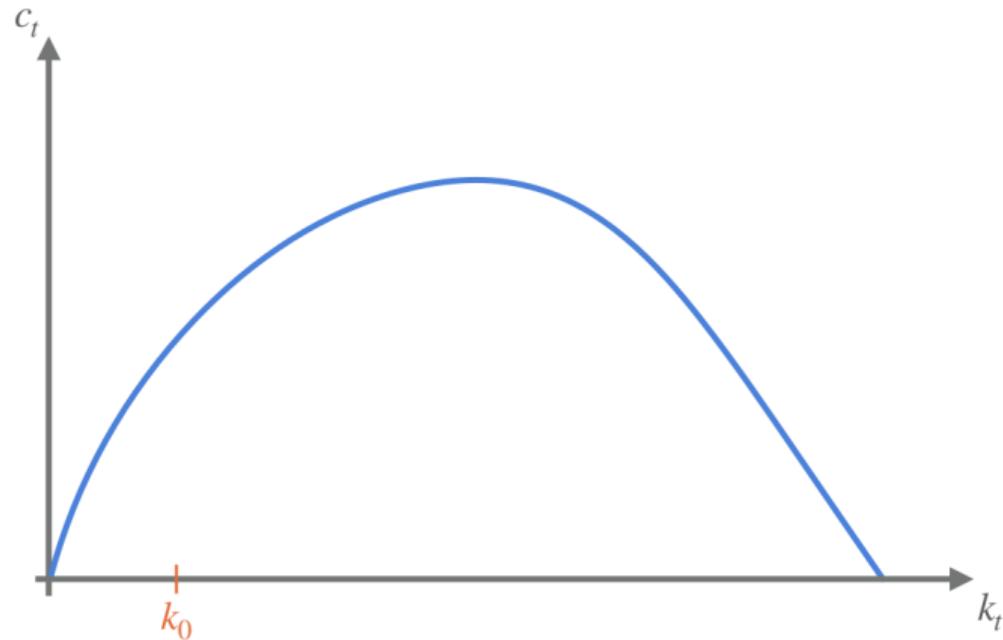
$$U'(c_t) = \beta \left[1 + F_k \underbrace{(F(k_t, 1) - c_t + (1 - \delta)k_t, 1) - \delta}_{k_{t+1}: \text{resource constraint}} \right] U'(c_{t+1})$$

Cannot explicitly link consumption across periods anymore—depends on k_t (thru r_t)!

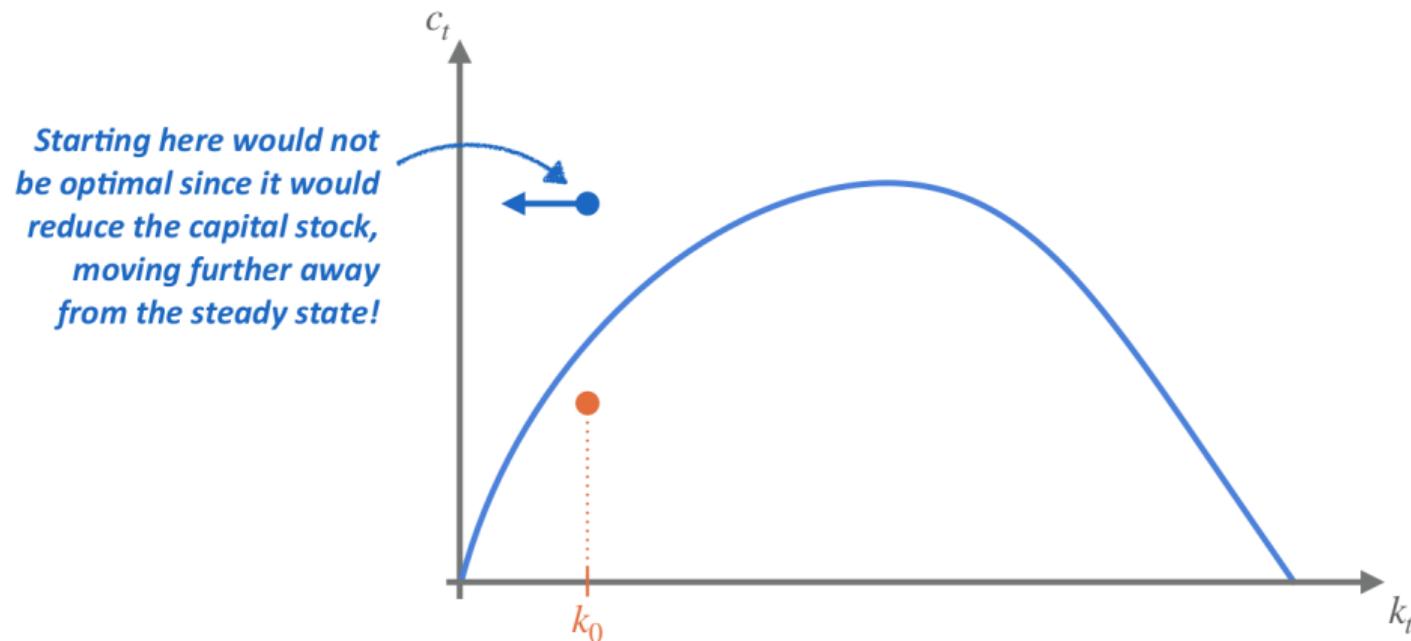
With **Cobb-Douglas** production and **log utility**...

$$c_{t+1} = \beta \left[1 + \alpha(k_t^\alpha - c_t + (1 - \delta)k_t)^{\alpha-1} - \delta \right] c_t$$

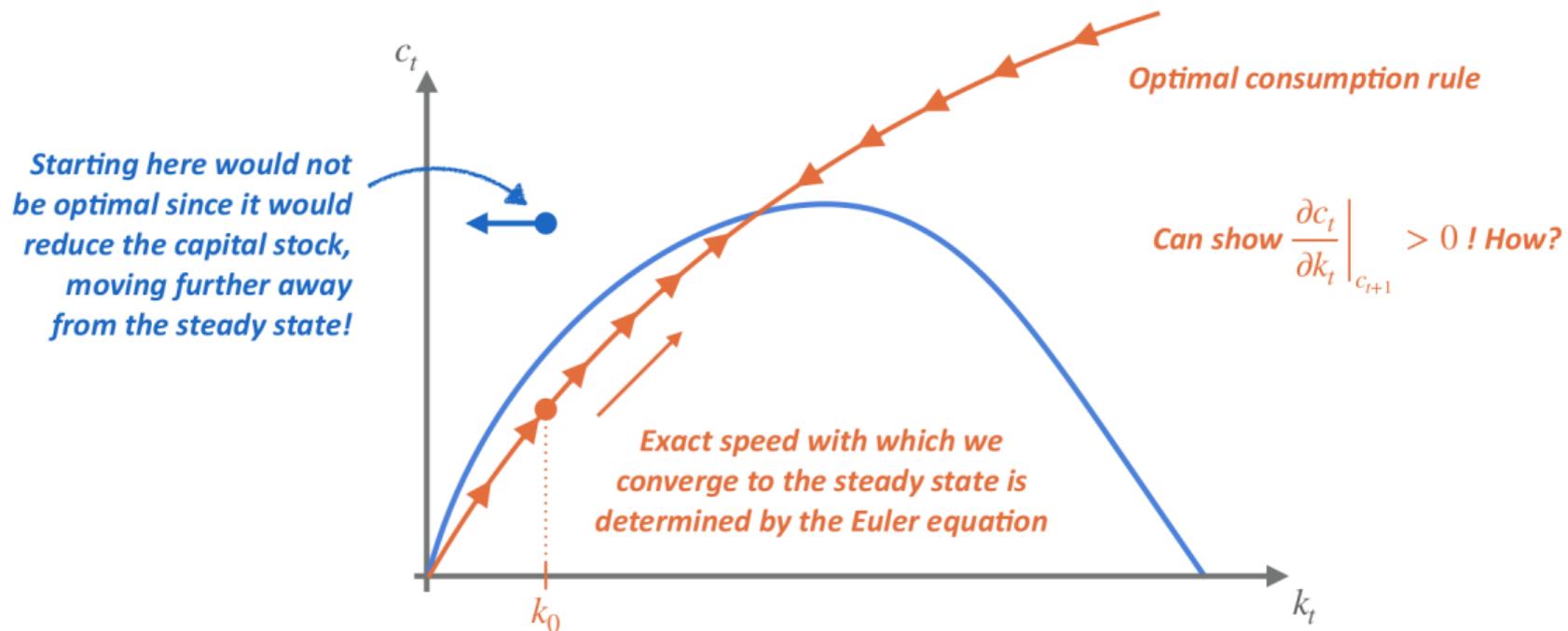
From any given k_0 , how do we transition to steady state?



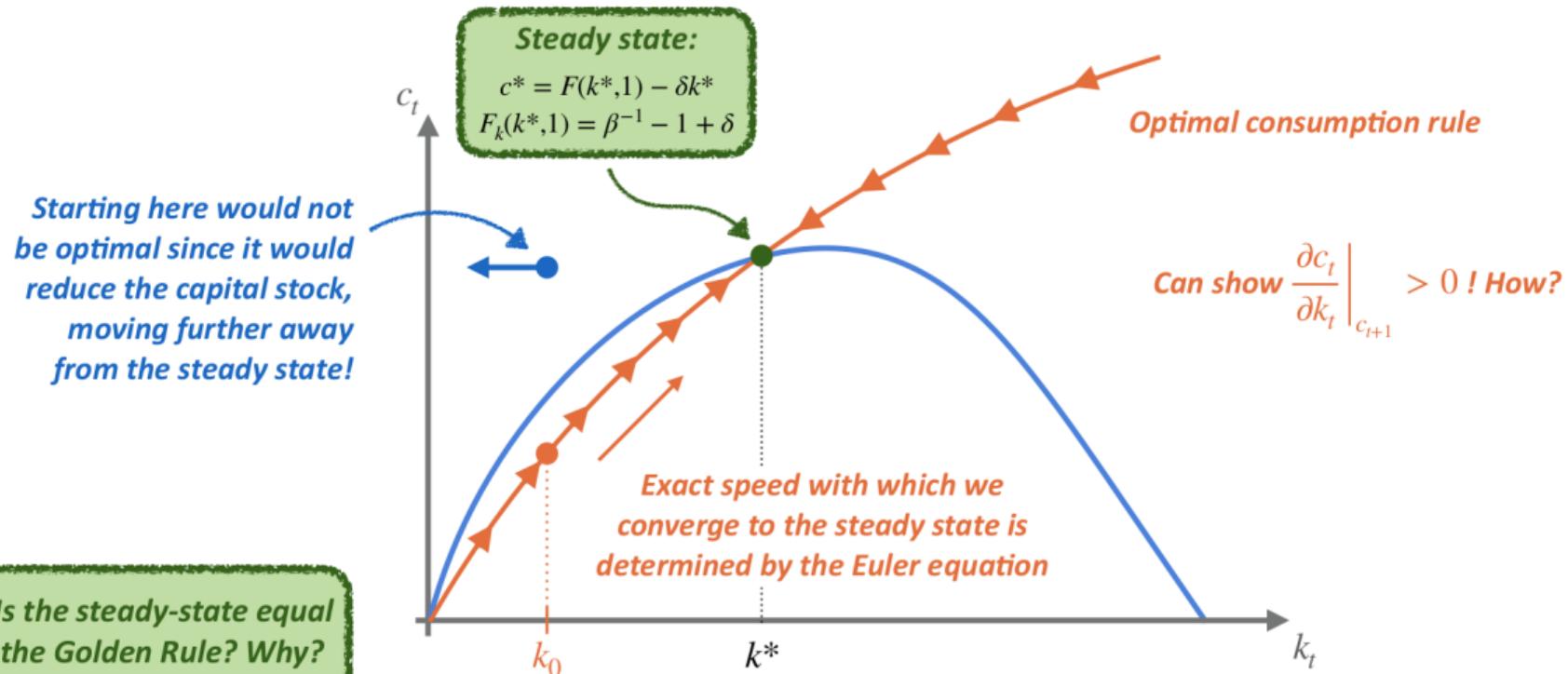
From any given k_0 , how do we transition to steady state?



From any given k_0 , how do we transition to steady state?



From any given k_0 , how do we transition to steady state?



Here's a special case we can solve in **closed-form**:

(Problem Set #3)

- Cobb-Douglas production function: $F(k_t, 1) = k_t^\alpha$
- Log utility: $U(c_t) = \ln(c_t)$
- Full depreciation: $\delta = 1$ *(pretty unrealistics unless we use long time periods)*

Then the **optimal consumption rule** is given by...

$$c_t = (1 - \alpha\beta)k_t^\alpha$$

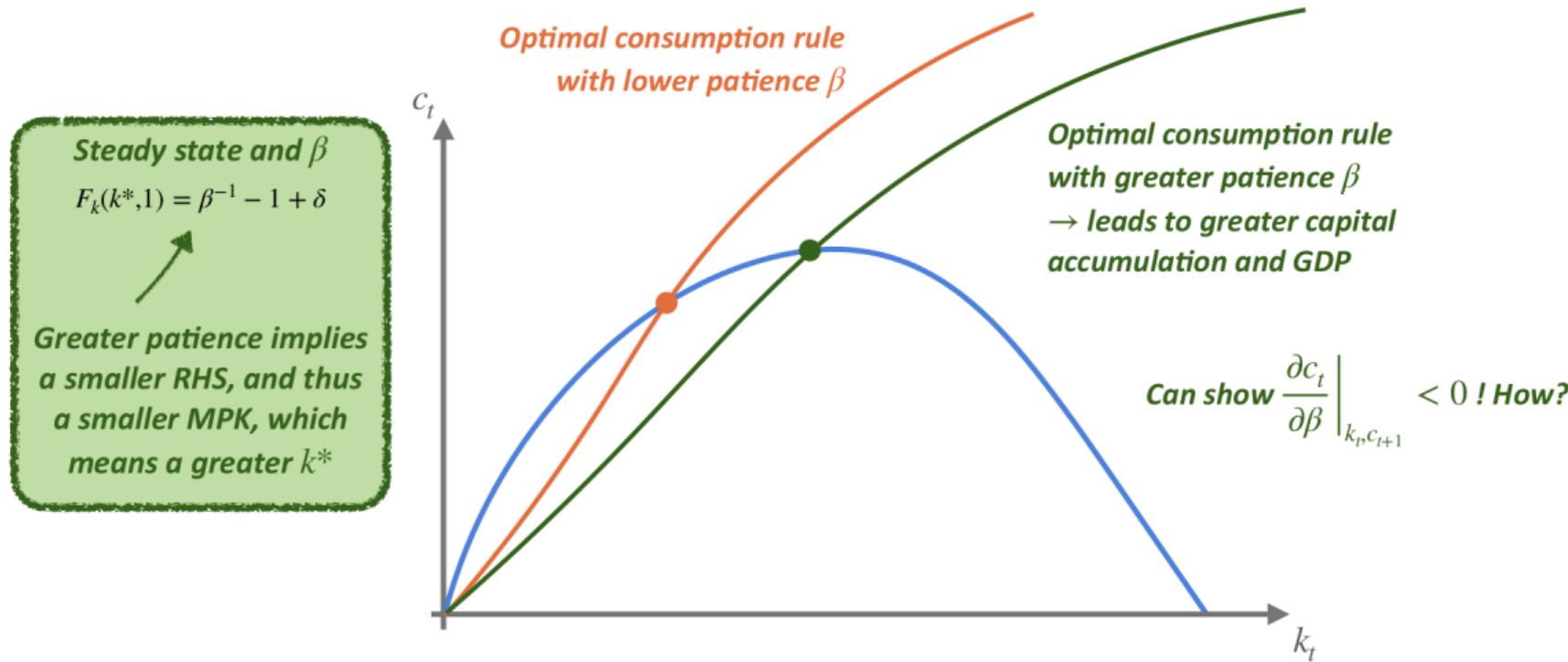
... which **looks exactly like Solow** just with **endogenous** savings rate $s = \alpha\beta$

Applications

Let's study **two applications** of the Ramsey-Cass-Koopmans model:

1. Compare two economies with different **discount factors**
 - *idea:* Protestant culture should lead to greater patience
 - *idea:* Aging economies should look like more patient economies
2. Introduce **capital taxes**
 - *idea:* Economies with extractive institutions should end up saving less

Application 1: Differences in discount factors



Application 2: Introduce capital taxes

- Think of capital taxes as **reducing the return on saving** in the Euler equation

$$U'(c_t) = \beta[1 + r_{t+1}(1 - \tau)]U'(c_{t+1}) \rightarrow (1 - \tau)[F_k(k^{ss}, 1) - \delta] = \frac{1}{\beta} - 1$$

- capital taxes ($\tau \uparrow$) have the **same effect** as less patience ($\beta \downarrow$)
- reduce capital accumulation and steady-state GDP per capita

The neoclassical growth model (NGM)

$$F(K_t, A_t L_t) = K_t^\alpha (A_t L_t)^{1-\alpha}, \quad K_{t+1} = \mathbf{F}(\mathbf{K}_t, \mathbf{A}_t \mathbf{L}_t) - \mathbf{C}_t + (1 - \delta) K_t$$

The neoclassical growth model (NGM)

$$F(K_t, A_t L_t) = K_t^\alpha (A_t L_t)^{1-\alpha}, \quad K_{t+1} = \mathbf{F}(\mathbf{K}_t, \mathbf{A}_t \mathbf{L}_t) - \mathbf{C}_t + (1 - \delta) K_t$$

... with **exogenous** savings (the **Solow** model)

- ad-hoc consumption rule → savings depend on current output y_t only

... with **endogenous** savings (the **Ramsey-Cass-Koopmans** model)

- optimal consumption path → savings depend on $\{w_t, r_t, \tau_t\}, \beta, k_0, \dots$
- can reincorporate population (g_L) and productivity (g_A) growth

How does the NGM answer our three organizing questions?

1. Why are we so rich and they so poor?
2. What is the engine of economic growth?
3. How do “growth miracles” happen?

How does the NGM answer our three organizing questions?

1. Why are we so rich and they so poor?

→ differences in s (β, τ, \dots), g_L , A_0

2. What is the engine of economic growth?

3. How do “growth miracles” happen?

How does the NGM answer our three organizing questions?

1. Why are we so rich and they so poor?

→ differences in s (β, τ, \dots), g_L , A_0

2. What is the engine of economic growth?

→ g_A , but exogenous!

3. How do “growth miracles” happen?

How does the NGM answer our three organizing questions?

1. Why are we so rich and they so poor?

→ differences in s (β, τ, \dots), g_L , A_0

2. What is the engine of economic growth?

→ g_A , but exogenous!

3. How do “growth miracles” happen?

→ transitional growth (faster if farther from BGP)

How does the NGM answer our three organizing questions?

1. Why are we so rich and they so poor?

→ differences in s (β, τ, \dots), g_L , A_0

2. What is the engine of economic growth?

→ g_A , but exogenous!

3. How do “growth miracles” happen?

→ transitional growth (faster if farther from BGP)

Next week: How do these fit the data?

Bonus: A better welfare metric?

- so far, we've compared economies i just by **GDP per capita** y_{it} . . .

“Every year, you get a check for $\$y_{it}$ (PPP-adjusted). . .”

Bonus: A better welfare metric?

- so far, we've compared economies i just by **GDP per capita** y_{it} . . .
"Every year, you get a check for \$ y_{it} (PPP-adjusted). . ."
- . . . but this week, we've shown how to **optimize consumption paths** c_{it}

"Every year, you choose to consume \$ c_{it} (PPP-adjusted). . ."

Bonus: A better welfare metric?

- so far, we've compared economies i just by **GDP per capita** y_{it} . . .

“Every year, you get a check for \$ y_{it} (PPP-adjusted). . .”

- . . . but this week, we've shown how to **optimize** consumption paths c_{it}

“Every year, you choose to consume \$ c_{it} (PPP-adjusted). . .”

- begs the question:* can we come up w/ a **better welfare metric** than just y_{it} ? say, one that incorporates . . .
 - optimal consumption-savings decisions?
 - optimal labor-leisure decisions?
 - mortality?
 - inequality?

*What proportion of consumption in the United States, given the US values of leisure, mortality, and inequality, would deliver **the same expected utility** as the values in country i ?*

*What proportion of consumption in the United States, given the US values of leisure, mortality, and inequality, would deliver **the same expected utility** as the values in country i ?*

We define **consumption-equivalent welfare** λ_i as the factor by which consumption in country i must be multiplied to yield the same expected lifetime utility as in the US:

$$U_{\text{US}}(\lambda_i) = U_i(1)$$

*What proportion of consumption in the United States, given the US values of leisure, mortality, and inequality, would deliver **the same expected utility** as the values in country i ?*

We define **consumption-equivalent welfare** λ_i as the factor by which consumption in country i must be multiplied to yield the same expected lifetime utility as in the US:

$$U_{\text{US}}(\lambda_i) = U_i(1)$$

Being a random person in country i (say, France) = Being a random person in the US with your consumption scaled by λ_i .

Suppose everyone in the world has the **same** preferences:

$$U(c, \ell) = \bar{u} + \ln(c) + v(\ell)$$

with, for each country i , no discounting ($\beta = 1$), no consumption growth ($g_c = 0$), constant leisure (ℓ_i), known life expectancy (e_i), and consumption distributed $\text{LogNormal}(c_i, \sigma_i^c)$ across people in i .

Suppose everyone in the world has the **same preferences**:

$$U(c, \ell) = \bar{u} + \ln(c) + v(\ell)$$

with, for each country i , no discounting ($\beta = 1$), no consumption growth ($g_c = 0$), constant leisure (ℓ_i), known life expectancy (e_i), and consumption distributed $\text{LogNormal}(c_i, \sigma_i^c)$ across people in i . Then **expected lifetime utility** in i is...

$$U_i = e_i [\bar{u} + \ln(c_i) + v(\ell_i) - 0.5\sigma_i^c]$$

Suppose everyone in the world has the **same preferences**:

$$U(c, \ell) = \bar{u} + \ln(c) + v(\ell)$$

with, for each country i , no discounting ($\beta = 1$), no consumption growth ($g_c = 0$), constant leisure (ℓ_i), known life expectancy (e_i), and consumption distributed $\text{LogNormal}(c_i, \sigma_i^c)$ across people in i . Then **expected lifetime utility** in i is...

$$U_i = e_i [\bar{u} + \ln(c_i) + v(\ell_i) - 0.5\sigma_i^c]$$

... and the (log) **consumption-equivalent welfare** in country i *relative to the US* is

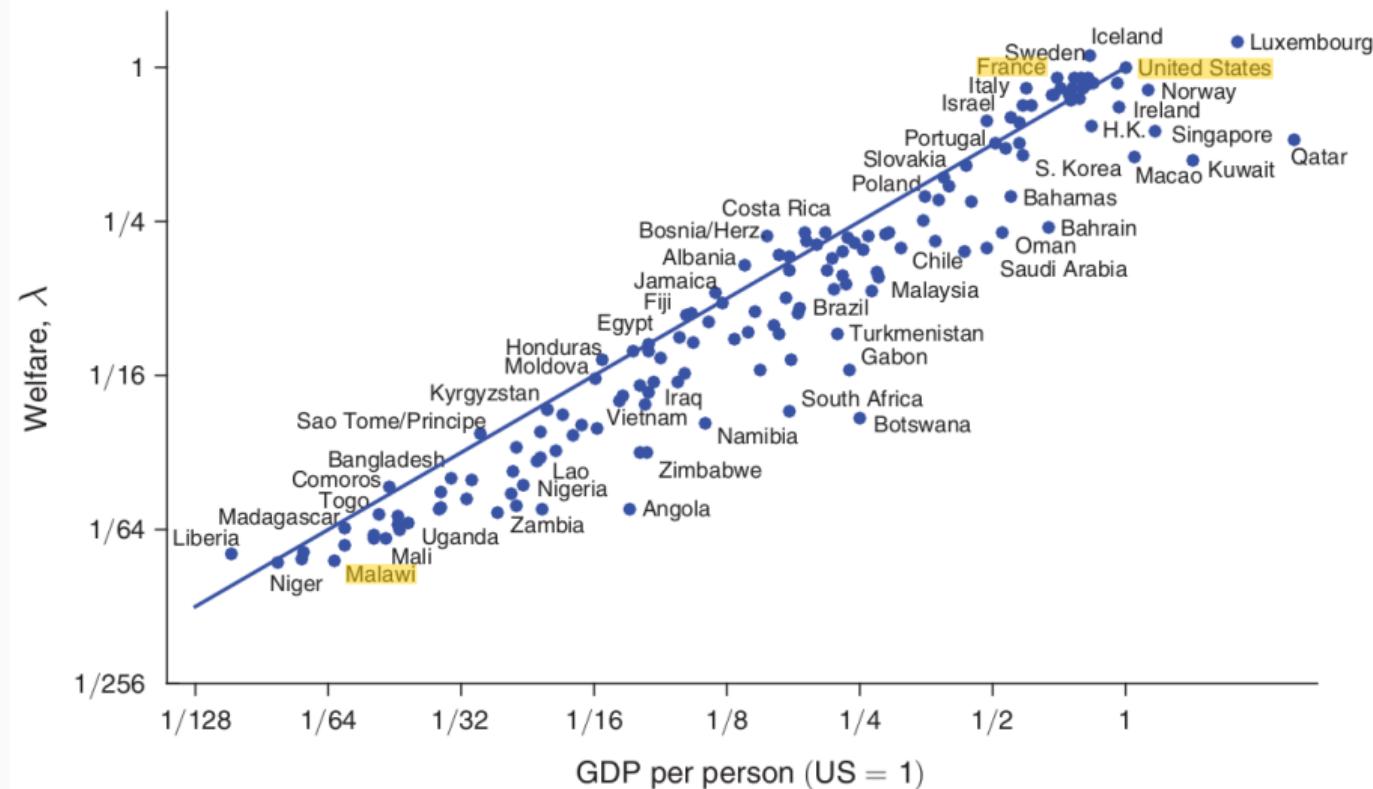
$$\ln \lambda_i = \frac{e_i - e_{\text{US}}}{e_{\text{US}}} [\bar{u} + \ln(c_i) + v(\ell_i) - 0.5\sigma_i^c] \quad (\text{Life expectancy})$$

$$+ \ln c_i - \ln c_{\text{US}} \quad (\text{Consumption})$$

$$+ v(\ell_i) - v(\ell_{\text{US}}) \quad (\text{Leisure})$$

$$- 0.5(\sigma_i^c - \sigma_{\text{US}}^c) \quad (\text{Inequality})$$

Panel A. Welfare and income are highly correlated at 0.96



Panel B. But this masks substantial variation in the ratio of λ to GDP per capita.
The mean absolute deviation from unity is about 27%

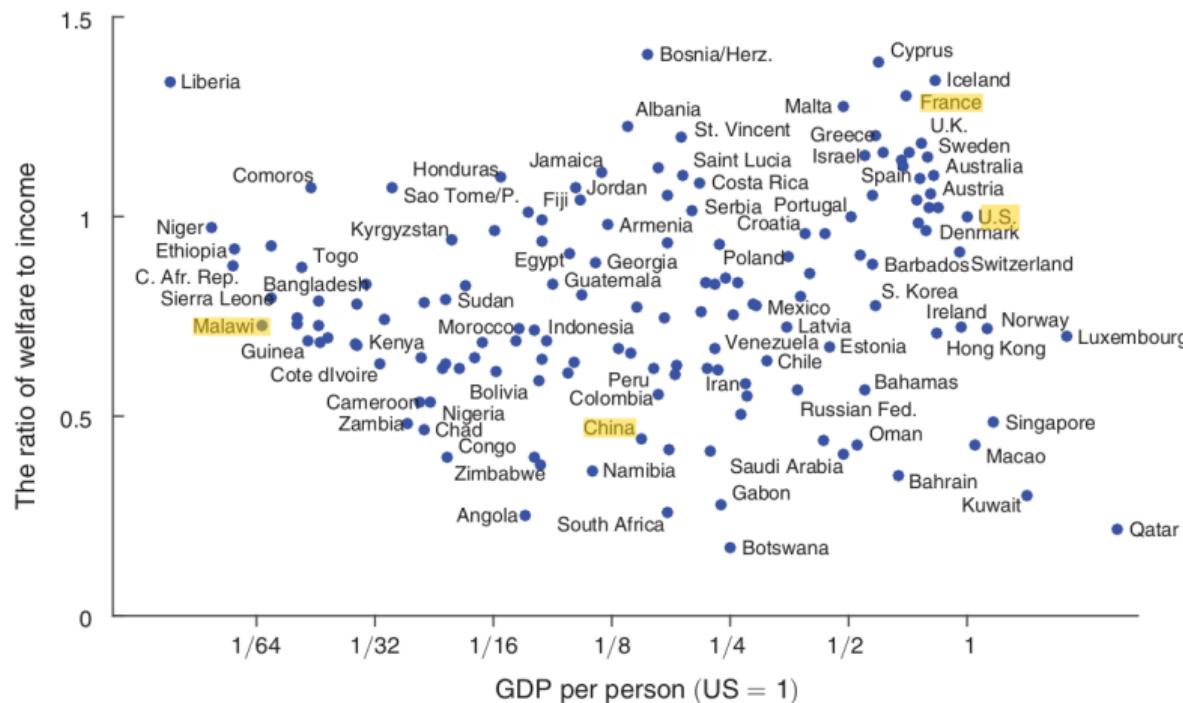


FIGURE 7. WELFARE USING MACRO DATA, 2007

...but still masks important variation

(Jones and Klenow, 2016)

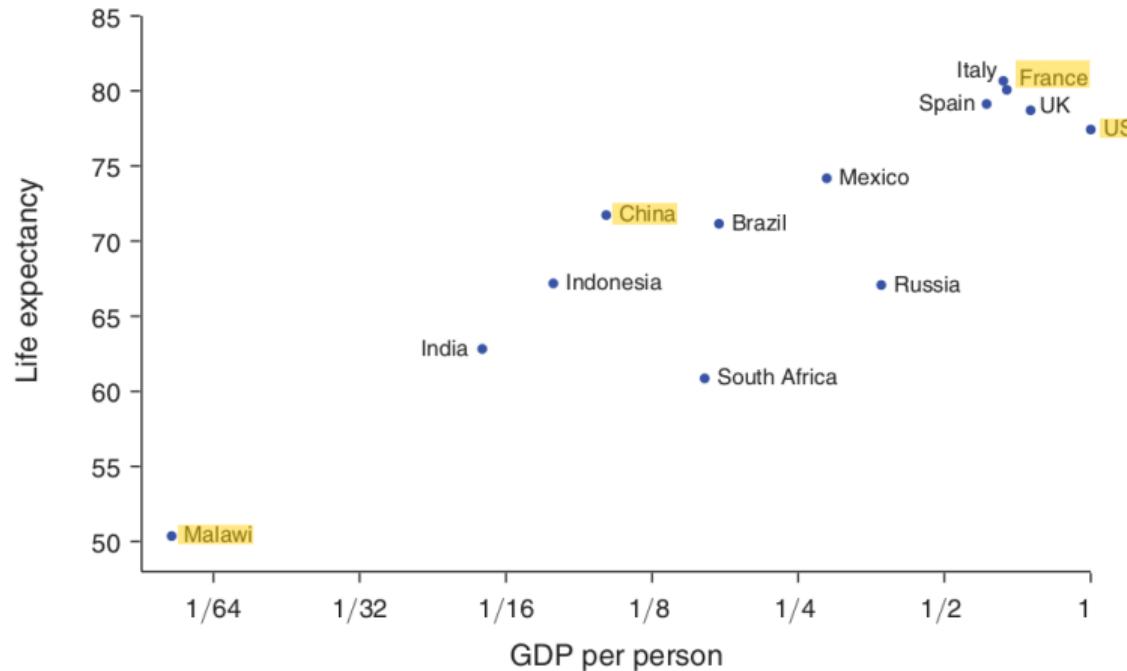


FIGURE 4. LIFE EXPECTANCY

Note: Life expectancy at birth in each country is measured as the sum over all ages of the probability of surviving to each age, using life tables from the World Health Organization.

... but still masks important variation

(Jones and Klenow, 2016)

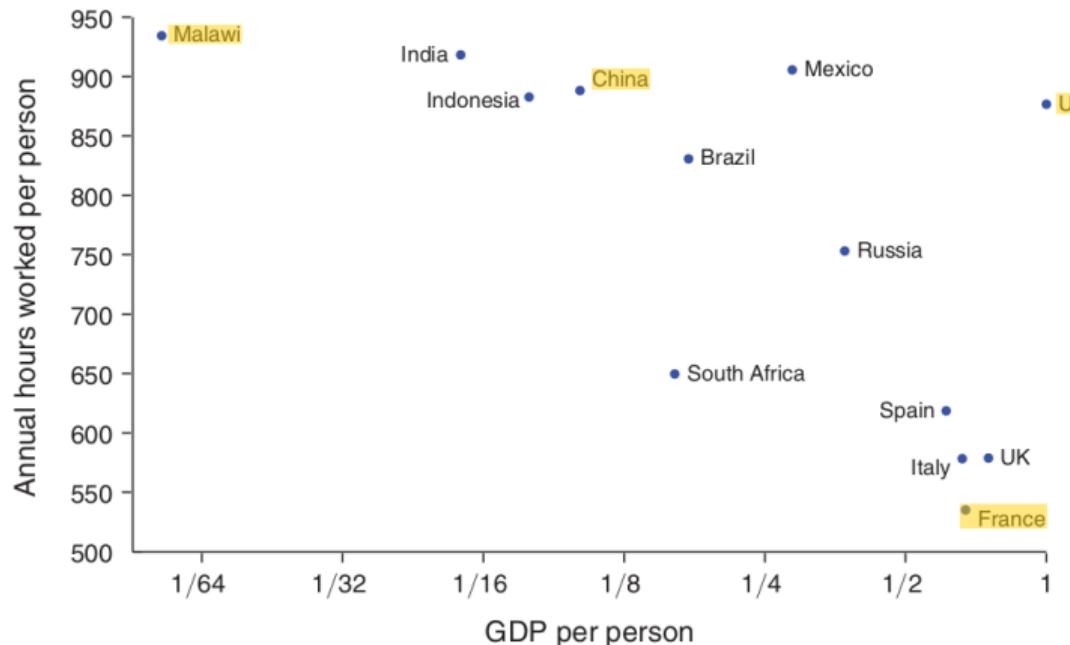


FIGURE 2. ANNUAL HOURS WORKED ACROSS COUNTRIES

Notes: The measure shown here of annual hours worked per capita is computed from the household surveys noted in Table 1, using survey-specific sampling weights and US survival rates across ages as in equation (16), with no time discounting.

... but still masks important variation

(Jones and Klenow, 2016)

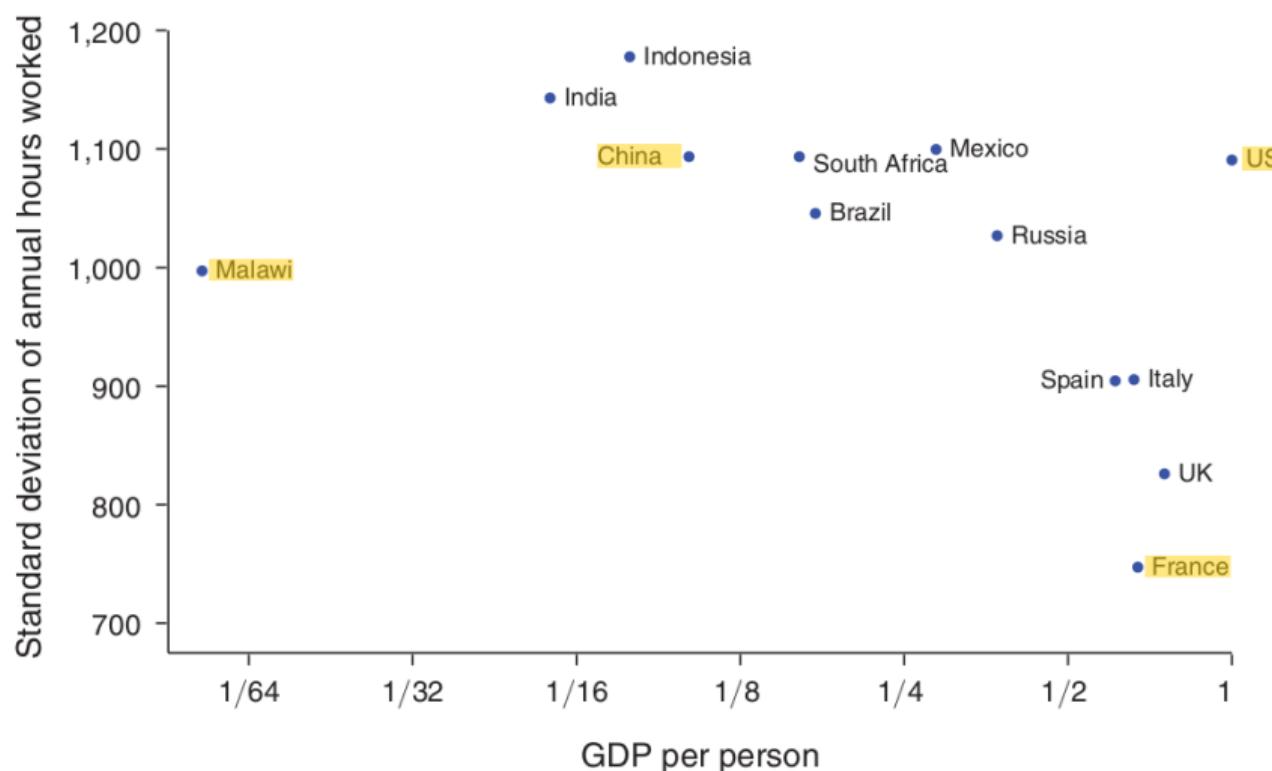


FIGURE 3. INEQUALITY IN ANNUAL HOURS WORKED

...but still masks important variation

(Jones and Klenow, 2016)

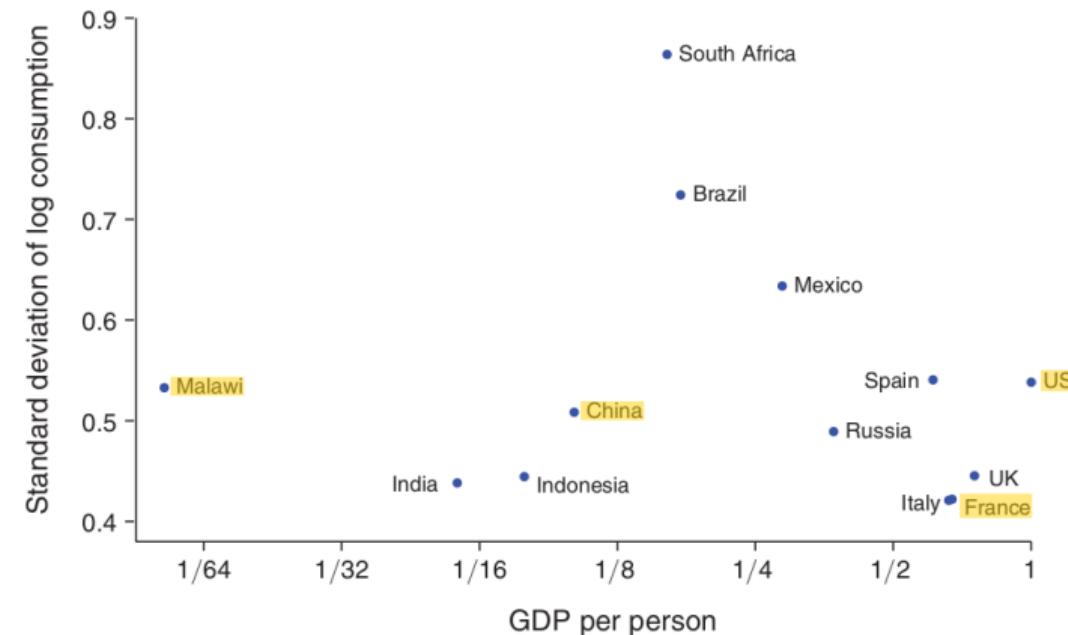


FIGURE 1. WITHIN-COUNTRY INEQUALITY

Notes: The standard deviation of log consumption within each economy is measured from the household surveys listed in Table 1. We use survey-specific sampling weights and US survival rates across ages using an analog of equation (17), with no discounting or growth.

That was for *levels*, but it's likewise for *growth*

(Jones and Klenow, 2016)

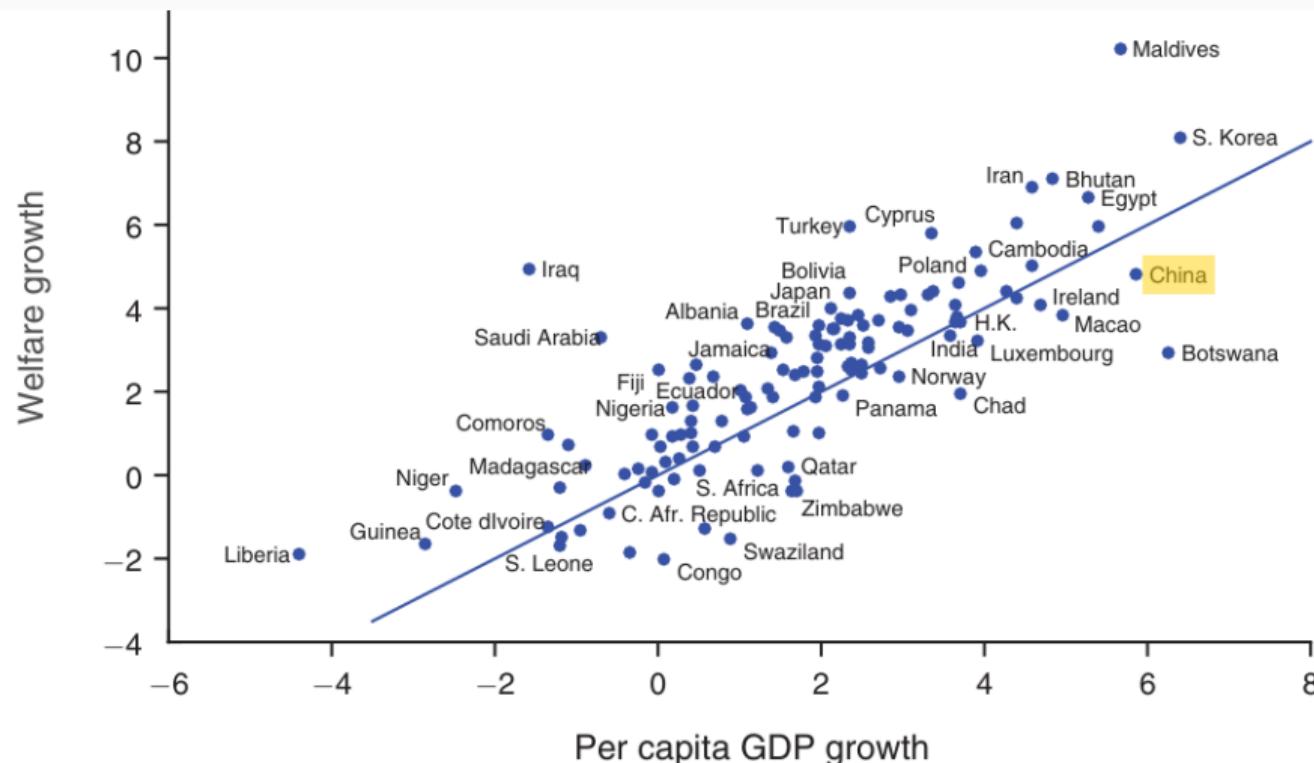
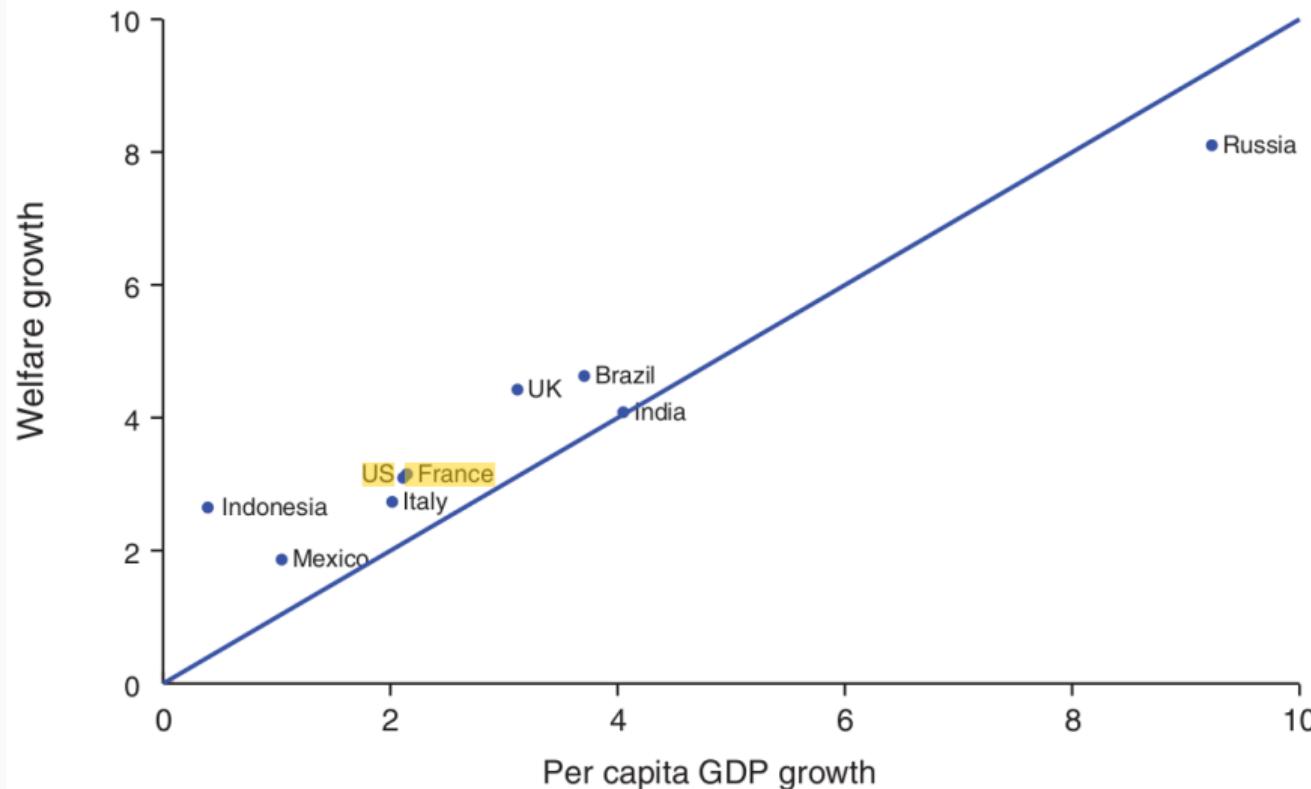


FIGURE 8. WELFARE AND INCOME GROWTH, 1980–2007 (Percent)

That was for *levels*, but it's likewise for *growth*

(Jones and Klenow, 2016)

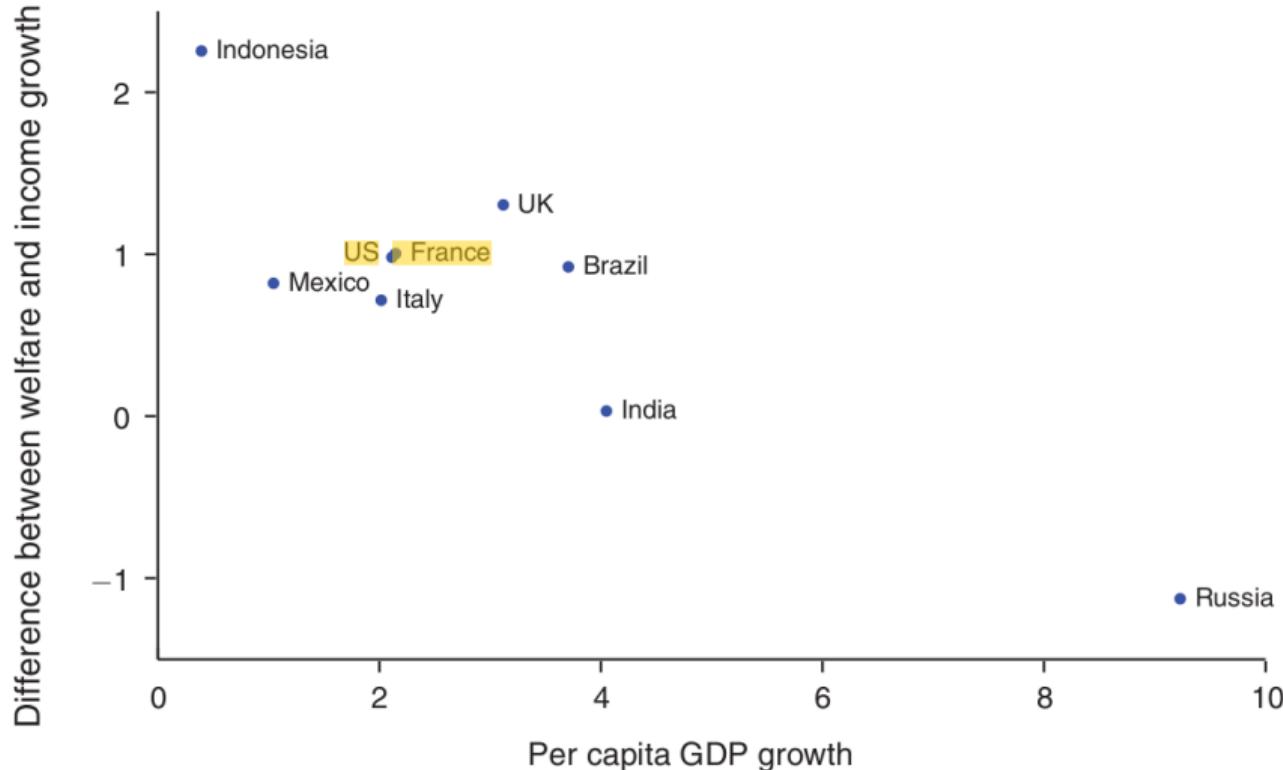
Panel A. The correlation between welfare growth and income growth is 0.97



That was for *levels*, but it's likewise for *growth*

(Jones and Klenow, 2016)

Panel B. The median absolute value of the difference between welfare and income growth is 0.95 percentage points



References

Barro, Robert J. and Xavier Sala-i Martin. 2004. *Economic growth*. Cambridge, MA: The MIT Press, second ed.

Brock, W. and L. Mirman. 1972. "Optimal growth under uncertainty: The discounted case." *Journal of Economic Theory* 4:479–513.

Jones, Charles I. and Peter J. Klenow. 2016. "Beyond GDP? Welfare across countries and time." *American Economic Review* 106 (9):2426–2457.

Kurlat, Pablo. 2020. *A course in modern macroeconomics*.

Phelps, Edmund. 1961. "The golden rule of accumulation: A fable for growthmen." *American Economic Review* 51 (4):638–643.