ECON 164: Theory of Economic Growth

Week 6: Ideas and Innovation

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Winter 2026

Recap

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  - **diminishing returns to capital** ($\alpha < 1$)
  - **neoclassical assumptions**: perfect competition, $F(\cdot, \cdot)$ is CRS w/ Inada conditions
  - **exogenous technological progress**: no modeled choices determine g_A

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  - Uzawa-Lucas model: $\dot{h} = \_\_h$ w/ CRS in $\{K, h\}$
  - pure externality model: $B = AK^{1-\alpha} \rightarrow$ collapses to vanilla AK

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- but, at best, innovation is **accidental** $\rightarrow$ what if it's **deliberate**?

Next two weeks

- focus on ideas $\rightarrow$ a **special kind** of economic good
    - "instructions for mixing raw materials together"
    - will require dropping **neoclassical assumptions**
- build (semi-)endogenous growth models w/ **costly**, **deliberate** idea accumulation
    - share aggregate dynamics in which A grows endogenously...
    - ...b/c firms invent **new** varieties of goods (Romer, 1990) [Week 6]
    - ...b/c firms invent **better** varieties of goods (Aghion and Howitt, 1992) [Week 7]
- **disclaimer**: these are models of "frontier" economies
    - discuss how technology diffuses to the rest of the world [Week 8]

The Idea Diagram

Ideas $\rightarrow$ Nonrivalry $\rightarrow$ Increasing returns $\rightarrow$ Imperfect competition

What is an idea?

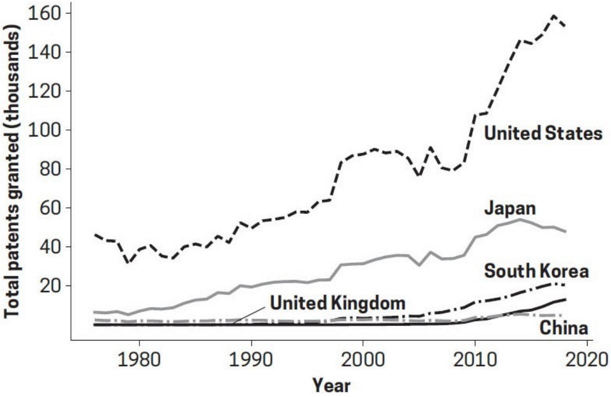
"instructions for mixing raw materials together" $\longrightarrow$ how to do **more** or **better** with **less**

- more and more-advanced ways to use tin (armor, pewter, ..., touch screens)
- pack double # transistors on computer chips every 18 mo. (Moore's Law)
- quality-adjusted price of light has fallen $4000\times$ since 1800 (Nordhaus, 1997)
- management practices (Walmart retail strategy, Ford assembly line)
- new goods (multiplex theater, diet soft drinks)

there are endless combinations! (Weitzman, 1998)

One way to see *some* idea accumulation

Figure 4.1 Patents Issued in the United States, by Country of Origin



SOURCE: OECD Patent Statistics (2014).

NOTE: These are counts of patents granted by the U.S. Patent Office, by the applicants' country of residence.

Ideas $\rightarrow$ Nonrivalry $\rightarrow$ ...

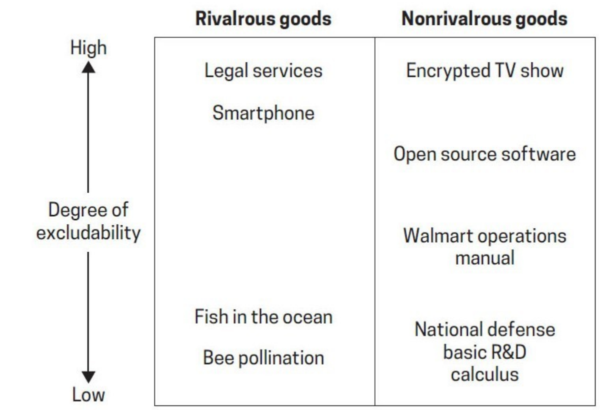
- Ideas are very different from factors of production (K, L) or regular goods
- **Factors and goods are rival...**
  - If I ate an apple, you cannot eat the same apple!
  - If you work an hour as a cook, you are not also driving an Uber that hour
  - If you use equipment to produce cars, cannot use the same to produce airplanes
- **...but ideas are nonrival**
  - If I learn how to fish, I can show you how to fish and we can both do it
  - If I learn *once* how to make soccer balls more efficiently, I can use it forever

Rivalry vs. excludability (Romer, 1992)

Ask yourself...

- **nonrival**: can one person's use of it diminish another's?
- **excludable**: can the owner charge a fee for its use?

Figure 4.4 Economic Attributes of Selected Products



SOURCE: This is a slightly modified version of Figure 1 from Romer (1993).

Ask yourself. . .

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Patents make nonrival ideas excludable!

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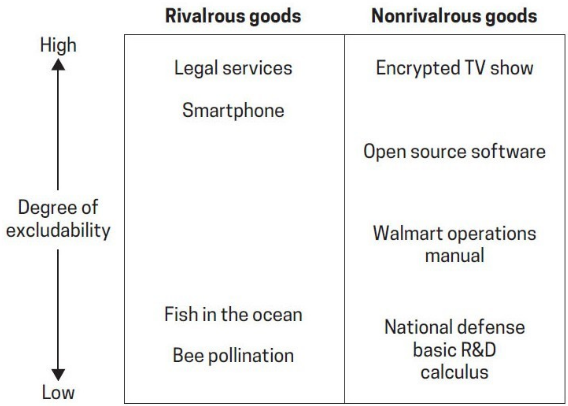

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Suppose our plant uses a standard neoclassical production function...

$$Y = F(A, K, L) = K^\alpha (AL)^{1-\alpha}$$

...which exhibits constant returns to scale in $\{K, L\}$

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. . . which exhibits constant returns to scale in $\{K, L\}$

If we want to double output, what must we replicate?

Build a new plant and populate it w/ identical machines (K) and workers (L), but. . .
. . . **the nonrival ideas (A) can be shared across plants!**

$$F(A, \lambda K, \lambda L) = \lambda Y \quad \rightarrow \quad F(\lambda A, \lambda K, \lambda L) > \lambda Y$$

If firms take prices as given in perfectly competitive markets. . .

$$
\begin{aligned}
\pi &= F(A, K, L) - rK - wL - fA \\
&= F(A, K, L) - \frac{\partial F}{\partial K}K - \frac{\partial F}{\partial L}L - \frac{\partial F}{\partial A}A \\
&= \underbrace{0}_{\text{b/c CRS in } \{K,L\}} - \frac{\partial F}{\partial A}A \\
&< 0
\end{aligned}
$$

. . . then they'd earn **negative profits** if they had to pay for $\boldsymbol{A}$

Fixed costs capture increasing returns + (need for) imperfect competition

Suppose you run a pharmaceutical company designing a new drug. . .

- you pay a **one-time cost** F to do the R&D
- you pay a **marginal cost** c to manufacture and package the pills

Your firm's average cost is

$$\text{AC} = \frac{cY + F}{Y} = \boldsymbol{c} + \frac{\boldsymbol{F}}{\boldsymbol{Y}} > c = \text{MC}$$

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You develop the drug if. . .

$$F \leq V = \underbrace{pY - cY}_{\text{operating profits}}$$

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You develop the drug if...

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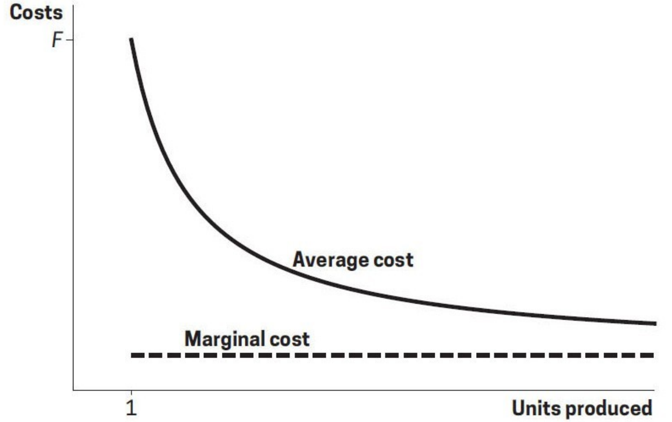
...but if perfect competition, $\boldsymbol{p = c}$ and thus $\boldsymbol{V = 0}$ $\rightarrow$ need some **market power**

Fixed costs capture increasing returns + imperfect competition

Figure 4.5 Costs Functions with Increasing Returns



NOTE: Average costs decline as production increases, but never fall below marginal cost, meaning the firm can never earn positive profits.

The Romer (1990) model

- won Paul Romer the 2018 Nobel Prize!
    - technically, this is the semi-endogenous version (Jones, 1995)
- embed the Idea Diagram in a general equilibrium model
- start with **aggregate dynamics** like we did with NGM, then **microfoundations**
    - aggregate dynamics are shared by many of these models, inc. next week's
    - microfoundation: firms invent **new** varieties of goods

Production, capital, and labor

The production function is (with Harrod-neutral productivity)

$$Y_t = K_t^{\alpha}(A_t \boldsymbol{s_Y} L_t)^{1-\alpha}$$

where $\boldsymbol{s_Y}$ is the share of labor allocated to **production**

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Capital and labor accumulation as in the **Solow** model:

$$\dot{K}_t = s_I Y_t - \delta K_t$$
$$\dot{L}_t = g_L L_t$$

R&D

For some parameters $\boldsymbol{\lambda} \in (0, 1]$ and $\boldsymbol{\phi} \in \mathbb{R}$, the accumulation of "ideas" obeys

$$\dot{A}_t = \theta(\boldsymbol{s_R} L_t)^{\boldsymbol{\lambda}} A_t^{\boldsymbol{\phi}}$$

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- $\boldsymbol{\lambda} < 1$: duplication of research effort, selection of worse scientists, ...

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- $\boldsymbol{\phi} > 0$: having ideas makes producing more ideas **easier**

  ("standing on the shoulders of giants" w/ calculus, CRISPR, ...)
- $\boldsymbol{\phi} < 0$: having ideas makes producing more ideas **harder**

  ("fishing out" best ideas, takes longer to reach research frontier)

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The original **Romer (1990)** model has $\boldsymbol{\lambda} = \phi = \mathbf{1}$, but we'll study the general case

later: what does the data imply re: $\boldsymbol{\lambda}$ and ϕ?

Growth of productivity

Divide both sides through by A_t to get...

$$g_A \equiv \frac{\dot{A}_t}{A_t} = \theta \boldsymbol{s_R}^{\boldsymbol{\lambda}} \left(\frac{L_t^{\boldsymbol{\lambda}}}{A_t^{1-\boldsymbol{\phi}}} \right)$$

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A **steady state** will be where g_A is constant $\longrightarrow \frac{L_t^{\lambda}}{A_t^{1-\phi}}$ is constant

how do we find it?

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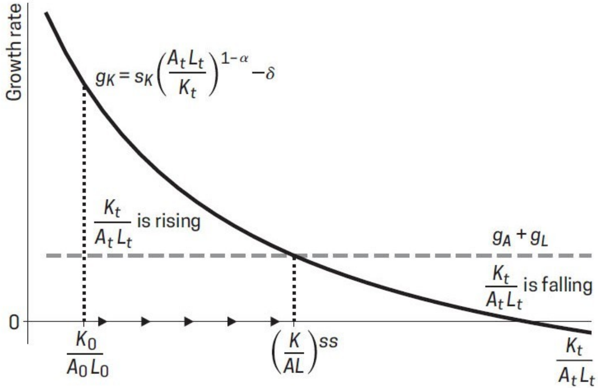
Recall from the Solow model a **similar type of expression**, but for g_K:

$$g_K \equiv \frac{\dot{K}_t}{K_t} = s_I \left(\frac{A_t L_t}{K_t} \right)^{1-\alpha} - \delta$$

Use a phase diagram

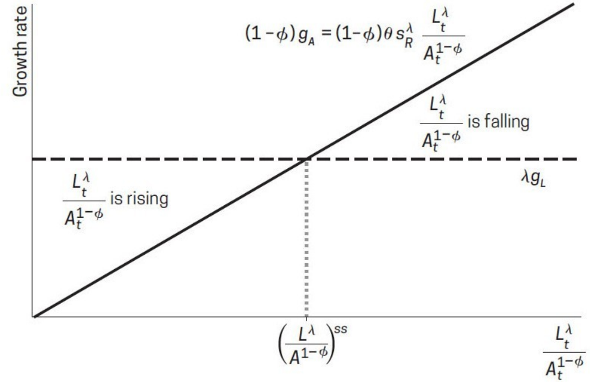
Figure 2.2 The Dynamics of the K_t/A_tL_t Ratio



NOTE: The dark line plots the growth rate of capital, g_K, from equation (2.11) against the K_t/A_tL_t ratio. The growth rate of AL, which is $g_A + g_L$, is plotted in the dashed line. Everywhere the dashed line is above the solid line, the K_t/A_tL_t ratio must be falling, as $g_K < g_A + g_L$. Everywhere the dashed line is below the solid line, the K_t/A_tL_t ratio must be rising, as $g_K > g_A + g_L$.

Use a phase diagram

Figure 5.1 The Dynamics of Productivity Growth



NOTE: The two curves plot the growth rate of the numerator and denominator of the ratio $L_t^\lambda / A_t^{1-\phi}$. The growth rate of the numerator is λg_L, plotted as the horizontal line. The growth rate of the denominator is $(1-\phi)g_A$, which from equation (5.8) is increasing in the ratio. The economy reaches a steady state where the two lines intersect, and at that point the growth rate of productivity is $g_A^{ss} = \frac{\lambda}{1-\phi} g_L$.

Characterizing the steady state

The steady-state growth rate of productivity is...

$$g_A^{\text{ss}} = \frac{\lambda}{1-\phi} g_L$$

...and the steady-state ratio is

$$\left(\frac{L^{\lambda}}{A^{1-\phi}}\right)^{\text{ss}} = \frac{g_A^{\text{ss}}}{\theta {s_R}^{\lambda}},$$

which means, along the BGP,

$$A_t^{\text{BGP}} = \left(\frac{\theta {s_R}^{\lambda}}{g_A^{\text{ss}}} L_t^{\lambda}\right)^{\frac{1}{1-\phi}}$$

Let $\tilde{k}_t \equiv K_t / (A_t(1-s_R)L_t) \to$ *notice* s_R*!*

Output per capita along the BGP is then

$$\begin{aligned} y_t^{\text{BGP}} &= \left(\tilde{k}^{\text{ss}}\right)^{\alpha} (1-s_R) A_t^{\text{BGP}} \\ &= \left(\tilde{k}^{\text{ss}}\right)^{\alpha} (1-s_R) \left(\frac{\theta {s_R}^{\lambda}}{g_A^{\text{ss}}}\right)^{\frac{1}{1-\phi}} L_t^{\frac{\lambda}{1-\phi}} \end{aligned}$$

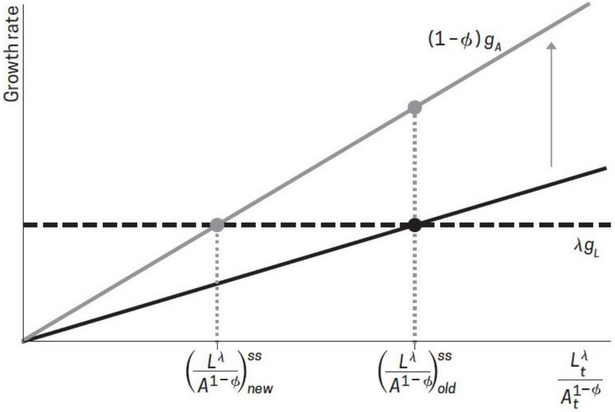
where, as in the Solow model,

$$\tilde{k}^{\text{ss}} = \left(\frac{s_I}{g_A^{\text{ss}} + g_L + \delta}\right)^{\frac{1}{1-\alpha}},$$

but now g_A^{ss} is **endogenous!**

An increase in s_R: What happens to $L_t^{\lambda}/A_t^{1-\phi}$?

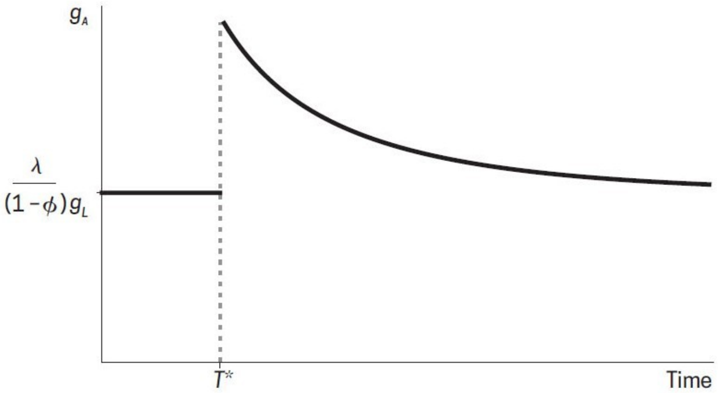
Figure 5.2 The Dynamics of an Increase in s_R



NOTE: An increase in s_R to s_R' rotates the g_A line upwards. In response, the growth rate g_A jumps up (the gray dot) and the ratio $L_t^\lambda/A_t^{1-\phi}$ declines to the new steady state. At that point the steady state growth rate of productivity is again $g_A^{ss} = \lambda g_L/(1-\phi)$.

An increase in s_R: What happens to g_A?

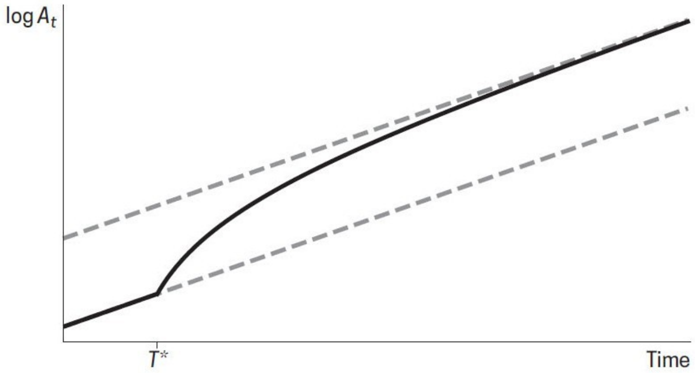
Figure 5.3 The Growth Rate of Productivity over Time



NOTE: At time T^* the share of researchers, s_R, increases. The growth rate spikes above the steady state value due to the increased production of ideas. But eventually the growth returns to the long-run value. Note that the steady state growth rate $g_A^{ss} = \lambda g_L/(1-\phi)$.

An increase in s_R: What happens to $\ln A_t$?

Figure 5.4 The Level of Productivity over Time



NOTE: At time T^* the share of researchers, s_R, increases. Because the growth rate g_A is temporariliy higher, the level of productivity, log A_t, increases rapidly and then remains along a higher path even as g_A falls back to the steady state growth rate. The dashed lines indicate the hypothetical paths of the lower and higher level of productivity the economy is moving between.

An increase in s_R: What happens to $\ln y_t$?

Recall...

$$\begin{aligned}\ln y_t &= \ln(1-s_R) + \ln A_t + \alpha \ln \tilde{k}_t \\ &= (1-\alpha)\left[\ln(1-s_R) + \ln A_t\right] + \alpha \ln k_t\end{aligned}$$

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. . . and therefore

$$\begin{aligned}g_y &= g_A + \alpha(g_K - g_A - g_L) \\ &= (1-\alpha)g_A + \alpha(g_K - g_L)\end{aligned}$$

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with, in the long run, ...

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So, at the time of the shock (T^*)...

- g_A immediately *jumps up*
- $\ln A_t$ does not immediately change
- $\ln y_t$ *drops* even though $\tilde{k}_t$ *jumps up*
  $\rightarrow g_K$ *drops* b/c eval'd at higher $\tilde{k}_t$

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...and in the long run

- $\ln y_t$ on *higher* BGP if s_R small enough

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But in between?

- generically, faster when farther
- but some ***drag*** from $\tilde{k}_t$ adjusting

What values for λ and ϕ can rationalize the data?

Consider the original Romer (1990) case: $\boldsymbol{\lambda} = \boldsymbol{\phi} = \mathbf{1}$ (like $\alpha = 1$ in Solow model)

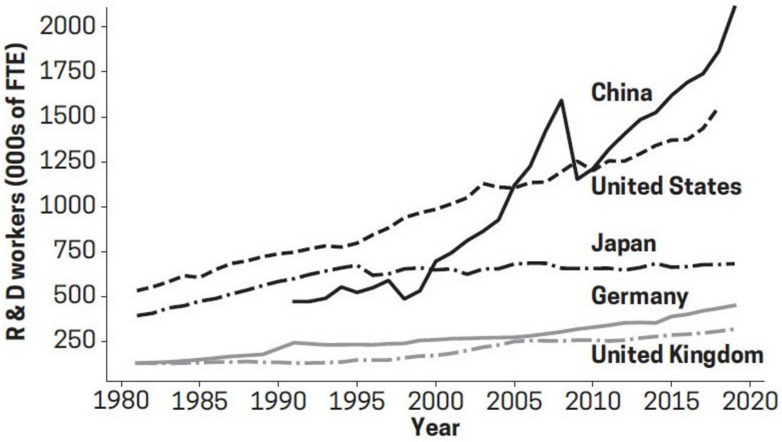
- characterization on Slide 16 doesn't work with $\phi = 1$

$$g_A^{\text{ss}} = \frac{\lambda}{1 - \phi} g_L$$

- instead, can show $\boldsymbol{g_A = \theta s_R L_t}$ → **"strong" scale effects** (like AK model)
    - growth rate of productivity depends on the **level** of population
    - growth would **explode** if population growth was positive
- can't just brute force eliminate them by respecifying as $\boldsymbol{g_A = \theta s_R}$
    - would suggest 10 people can produce as many ideas as 10mil if same s_R
    - in the data, even s_R has been growing!
- both imply a **growth effect** of changing s_R (just saw **level effect** when $\phi < 1$)

R&D employment *level* has been rising. . .

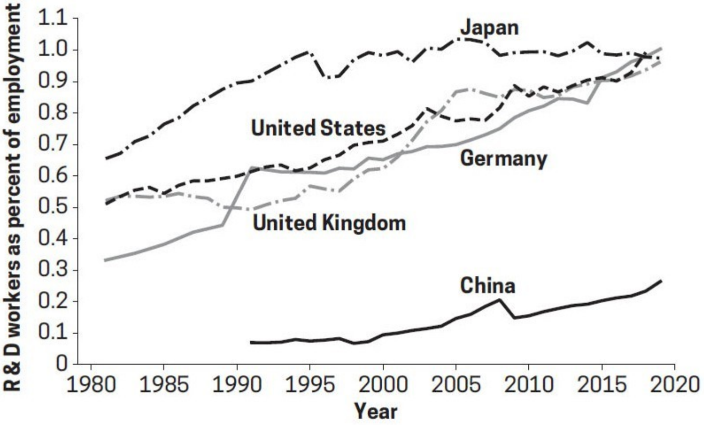
Figure 4.2 Number of R&D Workers (FTE), by Country



SOURCE: OECD Research and Development Statistics (2019).

. . . as has its *share*

Figure 4.3 R&D Workers as a Percent of Employment, by Country



SOURCE: Authors' calculations from OECD Research and Development Statistics (2019).

Instead, $\phi < 1$ implies "weak" scale effects

- cross-industry evidence suggests $\phi < 1$, even < 0 (Bloom et al., 2020)
- in that case, population **growth** is a **positive** for growth in output *per capita*

$$g_y^{\text{ss}} = \frac{\lambda}{1-\phi} g_L$$

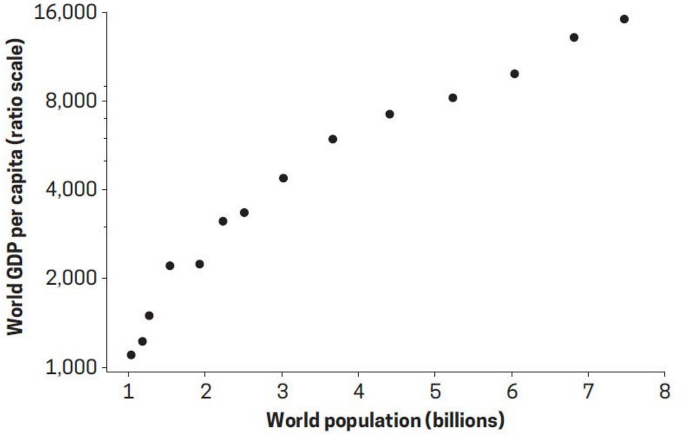
- **why?** more people don't dilute "ideas per person" because ideas are **nonrival**

(vs. capital in NGM)

- **but is that implication borne out in the data?**
  1. should only think about applying this model to the most advanced economies
  2. even among advanced economies, technology diffusion muddies the waters
  3. the model actually relates the growth in the *effective number of researchers* ($\boldsymbol{s_R L}$), rather than population, to economic growth $\longrightarrow$ more growth in former than latter

World population and world GDP per capita have moved hand-in-hand

Figure 4.6 World Population and GDP per Capita, 1820–2019



SOURCE: Authors' calculations from Bolt and van Zanden (2020).

Summary so far

- ideas are **special kind** of economic good

  Ideas $\rightarrow$ Nonrivalry $\rightarrow$ Increasing returns $\rightarrow$ Imperfect competition

- build (semi-)endogenous growth model w/ **costly**, **deliberate** idea accumulation
  - started w/ aggregate dynamics in which A grows endogenously...
  - **next**: what economic choices determine s_R in Romer (1990)?

An overview of production

There are two layers of production:

1. set of **final good** producers in a **perfectly competitive** market
   - think about Walmart, Target, ...
   - combine labor and intermediate goods to produce the final good
   - take prices as given (normalize output price to 1)
2. set of **intermediate good** producers in a **monopolistically competitive** market
   - sell the inventory, cash registers, buildings, etc. to Walmart
   - just use capital to produce each intermediate good (simplifying, not necessary)
   - each intermediate good is "small" but unique $\longrightarrow$ have some **market power**
   - will **choose** profit-maximizing price

The final goods sector

Assume a large number of identical firms using the same Cobb-Douglas technology. . .

$$Y_t = L_{Yt}^{1-\alpha} \int_0^{\boldsymbol{A_t}} x_{jt}^{\alpha} \, dj \qquad (L_{Yt} = (1 - \boldsymbol{s_R})L_t)$$

The final goods sector

Assume a large number of identical firms using the same Cobb-Douglas technology...

$$Y_t = L_{Yt}^{1-\alpha} \int_0^{\boldsymbol{A_t}} x_{jt}^{\alpha}\, dj \qquad (L_{Yt} = (1 - \boldsymbol{s_R})L_t)$$

... hire labor and intermediate goods to **maximize profits**:

$$\max_{L_{Yt}, \{x_{jt}\}} \; L_{Yt}^{1-\alpha} \int_0^{\boldsymbol{A_t}} x_{jt}^{\alpha}\, dj - wL_{Yt} - \int_0^{\boldsymbol{A_t}} p_{jt} x_{jt}\, dj,$$

which has first-order conditions

$$\boldsymbol{w_t} = (1-\alpha)\frac{Y_t}{L_{Yt}}, \qquad \boldsymbol{p_{jt}} = \alpha L_{Yt}^{1-\alpha} x_{jt}^{\alpha-1} \text{ for all } j \in [0, \boldsymbol{A_t}]$$

The intermediate goods sector

There are A_t monopolistically competitive firms, each of which…

- …produces its own unique good j using capital

$$x_{jt} = K_{jt}$$

- …chooses its **price** and **quantity** given its demand curve (from final goods producers)

$$p_{jt}(x_{jt}) = \alpha L_{Yt}^{1-\alpha} x_{jt}^{\alpha-1}$$

- …in order to **maximize (operating) profits**

$$\max_{x_{jt}} \ \pi_{jt} = p_{jt}(x_{jt})\, x_{jt} - r x_{jt}$$

Profit maximization for intermediate goods producer

The first-order condition is

$$\boldsymbol{r} = p_{jt} + x_{jt}\frac{\partial p_{jt}}{\partial x_{jt}} = p_{jt}\left(1 + \frac{x_{jt}}{p_{jt}}\frac{\partial p_{jt}}{\partial x_{jt}}\right) = p_{jt}\left(1 + (\alpha - 1)\right) = \boldsymbol{\alpha p_{jt}}$$

So, all intermediate producers charge the same **markup** over marginal cost

$$p_{jt} \equiv \boldsymbol{p_t = \frac{1}{\alpha}}\boldsymbol{r}$$

Profit maximization for intermediate goods producer

The first-order condition is

$$\boldsymbol{r} = p_{jt} + x_{jt}\frac{\partial p_{jt}}{\partial x_{jt}} = p_{jt}\left(1 + \frac{x_{jt}}{p_{jt}}\frac{\partial p_{jt}}{\partial x_{jt}}\right) = p_{jt}\left(1 + (\alpha - 1)\right) = \boldsymbol{\alpha p_{jt}}$$

So, all intermediate producers charge the same **markup** over marginal cost

$$p_{jt} \equiv \boldsymbol{p_t = \frac{1}{\alpha} r}$$

Intuition: the **price elasticity of demand** is...

$$\frac{\partial \ln x_{jt}}{\partial \ln p_{jt}} = \frac{p_{jt}}{x_{jt}}\frac{\partial x_{jt}}{\partial p_{jt}} = \boldsymbol{-\frac{1}{1-\alpha}}$$

...so $\alpha \to 1$ looks like perfect competition!

Adding up production

Intermediate producers set the same price $\rightarrow$ choose the same quantity $\boldsymbol{x_{jt} = x_t}$

$$\int_0^{\boldsymbol{A_t}} x_{jt}\, dj = K_t \quad \rightarrow \quad x_t = \frac{K_t}{\boldsymbol{A_t}}$$

Adding up production

Intermediate producers set the same price $\rightarrow$ choose the same quantity $\boldsymbol{x_{jt} = x_t}$

$$\int_0^{\boldsymbol{A_t}} x_{jt}\, dj = K_t \quad \rightarrow \quad x_t = \frac{K_t}{\boldsymbol{A_t}}$$

Substituting this back into the final goods production function...

$$Y_t = L_{Yt}^{1-\alpha} \boldsymbol{A_t} x_t^{\alpha} = L_{Yt}^{1-\alpha} \boldsymbol{A_t} \left(\frac{K_t}{\boldsymbol{A_t}}\right)^{\alpha} = K_t^{\alpha} \left(\boldsymbol{A_t} L_{Yt}\right)^{1-\alpha}$$

...which is where we started before introducing the microfoundation!

Profits and their growth

How much do final goods producers **spend** on intermediates?

$$
\begin{aligned}
0 = \pi_t^{\text{final}} &= Y_t - wL_{Yt} - \boldsymbol{A_t} p_t x_t \\
&= Y_t - \left((1-\alpha)\frac{Y_t}{L_{Yt}} \right) L_{Yt} - \boldsymbol{A_t} p_t x_t \\
&= \alpha Y_t - \boldsymbol{A_t} p_t x_t \\
p_t x_t &= \alpha \frac{Y_t}{\boldsymbol{A_t}}
\end{aligned}
$$

How much do intermediate goods producers **earn** in operating profits?

$$\boldsymbol{\pi_t} = p_t x_t - r_t x_t = (p_t - \alpha p_t) x_t = (1-\alpha) p_t x_t = \boldsymbol{(1-\alpha)\alpha \frac{Y_t}{A_t}}$$

It follows that along a BGP...

$$\boldsymbol{g_\pi} = g_Y - g_A = (g_A + g_L) - g_A = \boldsymbol{g_L}$$

The R&D decision

- Assume intermediate good producers are "small" $\rightarrow$ take $\dot{A}$ as given
- Pay a **fixed cost** at t in units of labor to create one new idea:

$$F_t = w_t \frac{s_R L_t}{\dot{A}_t} = \frac{w_t}{\theta} (s_R L_t)^{1-\lambda} A_t^{-\phi}$$

- The **value** of a new idea at t is the **PDV of profits** it earns:

$$V_t = \int_t^\infty e^{-r\tau} \left(e^{g_\pi \tau} \pi_t\right) \, d\tau = \frac{\pi_t}{r - g_\pi}$$

- A (potential) producer will try to create a new idea if $\boldsymbol{F_t \leq V_t}$
- **Free entry** of potential producers $\rightarrow$ competition ensures $\boldsymbol{F_t = V_t}$ in equilibrium

Putting it all together to pin down s_R on BGP

Start from the free entry condition $F_t = V_t$:

$$\frac{\pi_t}{r - g_\pi} = w_t \frac{s_R L_t}{\dot{A}_t}$$

Now substitute expressions for π_t, g_π, w_t, and $\dot{A}_t$ from previous slides:

$$\frac{\alpha(1-\alpha)\frac{Y_t}{A_t}}{r - g_L} = \left[(1-\alpha)\frac{Y_t}{(1-s_R)L_t}\right]\frac{s_R L_t}{g_A A_t}$$

Rearrange and cancel terms:

$$\frac{s_R}{1-s_R} = \frac{\alpha(1-\alpha)}{1-\alpha}\frac{g_A}{r-g_L} \quad \rightarrow \quad s_R = \frac{1}{1+\frac{r-g_L}{\alpha g_A}}$$

(r, g_A are endogenous but independent of s_R on BGP)

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