ECON 164: Theory of Economic Growth

Week 7: Ideas and Innovation (II)

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Winter 2026

Reminder of what we're doing

- focus on ideas → a **special kind** of economic good
  - "instructions for mixing raw materials together"
  - will require dropping **neoclassical assumptions**
- build (semi-)endogenous growth models w/ **costly**, **deliberate** idea accumulation
  - share aggregate dynamics in which A grows endogenously...
  - ...b/c firms invent **new** varieties of goods (Romer, 1990) [Week 6]
  - ...b/c firms invent **better** varieties of goods (Aghion and Howitt, 1992) [Week 7]
- **disclaimer**: these are models of "frontier" economies
  - discuss how technology diffuses to the rest of the world [Week 8]

The Schumpeterian model

- pioneered by Aghion and Howitt (1992) and Grossman and Helpman (1991)
- sometimes called "quality ladder" or "creative destruction" models
  - won Aghion & Howitt the 2025 Nobel Prize!
  - technically, this is the semi-endogenous version (Segerstrom, 1998)
- another way to embed the Idea Diagram in a general equilibrium model
- start with **aggregate dynamics** like we did with NGM, then **microfoundations**
  - aggregate dynamics are **identical** to those in Romer (1990)
  - microfoundation: firms now invent **better** varieties of goods

Production, capital, and labor

The production function is (with Harrod-neutral productivity)

$$Y_t = K_t^{\alpha}(A_t \boldsymbol{s_Y} L_t)^{1-\alpha}$$

where $\boldsymbol{s_Y}$ is the share of labor allocated to **production**

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next few slides: treat $\boldsymbol{s_R}$ as given and constant

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Capital and labor accumulation as in the **Solow** model:

$$\dot{K}_t = s_I Y_t - \delta K_t$$
$$\dot{L}_t = g_L L_t$$

R&D

For some parameters $\boldsymbol{\lambda} \in (0,1]$, $\phi \in \mathbb{R}$, and $\gamma > 0$, the accumulation of ideas obeys

$$A_t = (1+\gamma)^{\boldsymbol{N_t}} \quad \text{with} \quad \mathbb{E}\left[\dot{\boldsymbol{N}}_{\boldsymbol{t}}\right] = \theta \frac{(\boldsymbol{s_R} L_t)^{\boldsymbol{\lambda}}}{A_t^{1-\phi}}$$

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- **ideas are "steps" up the "quality ladder"**
  - $\boldsymbol{N_t}$ is the *number* of steps taken by t ("how *many* new versions have there been?")
  - γ is the *size* of each step ("how much *better* is each new version?")
  - steps are *discrete* $\longrightarrow$ R&D raises the *probability* of a step (hence $\mathbb{E}$)

(formally, $\boldsymbol{N_t}$ is a Poisson process)

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    (formally, $\boldsymbol{N_t}$ is a Poisson process)
- $\boldsymbol{\lambda}$ and ϕ have the same interpretation as before

Growth of productivity

First take logs and use that $\ln(1+x) \approx x$ for small x...

$$\ln A_t = \boldsymbol{N_t} \ln(1+\gamma) \approx \gamma \boldsymbol{N_t}$$

...then take time derivatives and expectations to get

$$\mathbb{E}[g_A] \equiv \mathbb{E}\left[\frac{d \ln A_t}{dt}\right] = \gamma \mathbb{E}\left[\dot{\boldsymbol{N}}_{\boldsymbol{t}}\right] = \gamma \theta \boldsymbol{s_R}^{\boldsymbol{\lambda}} \left(\frac{L_t^{\boldsymbol{\lambda}}}{A_t^{1-\phi}}\right)$$

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Look familiar?

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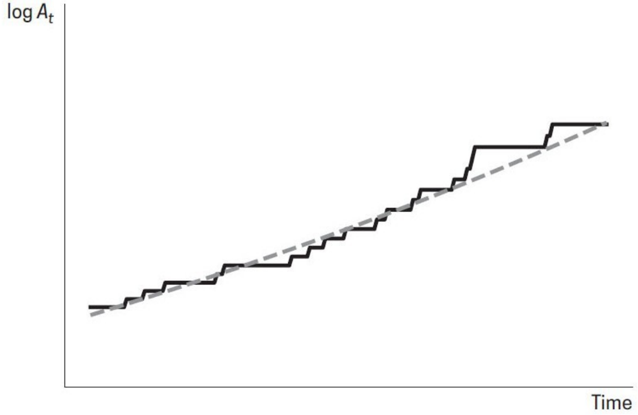
Look familiar? Same as Romer (1990), but for two differences...

- γ: no big deal, just acts like Romer (1990) with $\tilde{\theta} \equiv \gamma\theta$
- $\mathbb{E}$: realized path of A_t will be choppy, but expected path is smooth

Realized vs. expected path of productivity

Figure 6.1 Productivity Growth in the Schumpeterian Model



NOTE: The black line plots an actual path of productivity in the Schumpeterian model. Because innovations arrive randomly, productivity is flat for some periods, before jumping discretely by the amount γ. The gray dashed line plots $E[\ln A]$, or the path of productivity in expectation.

Characterizing the steady state

A **steady state** will be where $\mathbb{E}[g_A]$ is constant $\rightarrow \frac{L_t^{\lambda}}{A_t^{1-\phi}}$ is constant

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Use the **same phase diagram** as before! Will find (again)...

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$$\mathbb{E}[g_A]^{\text{ss}} = \frac{\lambda}{1-\phi} g_L \quad \longrightarrow \quad \gamma \textit{ doesn't show up!}$$

Why not? With bigger γ, **larger steps vs. less frequent steps**:

$$\mathbb{E}\left[\dot{N}\right]^{\text{ss}} = \frac{\mathbb{E}[g_A]^{\text{ss}}}{\gamma} = \frac{\lambda}{1-\phi}\frac{g_L}{\gamma}$$

Summary so far

Building *another* (semi-)endogenous growth model w/ **costly**, **deliberate** R&D

- same aggregate dynamics as in Romer (1990)...
  - $\frac{L_t^{\lambda}}{A_t^{1-\phi}}$ must be constant on BGP → use **phase diagram** to characterize
  - growth rate of productivity (in expectation) is again $\mathbb{E}[g_A]^{\text{ss}} = \frac{\lambda}{1-\phi} g_L$
  - same comparative statics, weak scale effects, level effects, etc.
- ...but now it's not *new* goods, but discretely **better** (γ) goods driving $\boldsymbol{A_t}$
- **next**: what economic choices determine $\boldsymbol{s_R}$ in the Schumpeterian model?
  - firms will now anticipate **losing** their monopoly power when something better arrives

An overview of production

Again, there are two layers of production:

1. set of **final good** producers in a perfectly competitive market
    - think about Walmart, Target, . . .
    - combine labor and **chosen vintage** of intermediate good to produce the final good
    - take prices as given (normalize output price to 1)
2. set of **intermediate good** producers can engage in Bertrand competition
    - sell, e.g., the inventory management software that Walmart uses
    - just use capital to produce each intermediate good (simplifying, not necessary)
    - compete by **setting prices** s.t. differences in quality across vintages, but. . .
    - . . . under reasonable assumptions, only the **currently-best vintage** will ever be sold

$\rightarrow$ ***current market leader gets to act like a monopolist***

The final goods sector

Assume a large number of identical firms using the same Cobb-Douglas technology...

$$Y_t = L_{Yt}^{1-\alpha} A_{jt}^{1-\alpha} x_{jt}^{\alpha} \qquad (L_{Yt} = (1 - \boldsymbol{s_R}) L_t)$$

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...hire labor and **some vintage $\boldsymbol{j}$** of the intermediate good to **maximize profits**:

$$\max_{L_{Yt}, x_{\boldsymbol{j}t}} \; L_{Yt}^{1-\alpha} A_{\boldsymbol{j}t}^{1-\alpha} x_{\boldsymbol{j}t}^{\alpha} - wL_{Yt} - p_{\boldsymbol{j}t} x_{\boldsymbol{j}t},$$

which has first-order conditions

$$\boldsymbol{w_t} = (1 - \alpha)\frac{Y_t}{L_{Yt}}, \qquad \boldsymbol{p_{jt}} = \alpha L_{Yt}^{1-\alpha} A_{jt}^{1-\alpha} x_{jt}^{\alpha-1} \text{ for chosen } \boldsymbol{j}$$

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(would pay more for a better vintage!)

The intermediate goods sector

There are **potentially many** firms, each of which, **if chosen**, would...

- ... produce its own unique vintage j using capital

$$x_{jt} = K_{jt}$$

- ... choose its **price** and **quantity** given its demand curve (possibly strategically)

$$p_{jt}(x_{jt}) = \alpha L_{Yt}^{1-\alpha} A_{jt}^{1-\alpha} x_{jt}^{\alpha-1}$$

- ... in order to **maximize (operating) profits**

$$\max_{x_{jt}} \; \pi_{jt} = p_{jt}(x_{jt})\, x_{jt} - r x_{jt}$$

Which vintage j is chosen?

Let's think about how the top two firms, N and $N-1$, would compete...

- Bertrand competition: set prices **simultaneously** and **strategically**
- For the same quantity demanded, the **better** vintage can set a higher price:

$$\frac{p_N}{p_{N-1}} = \frac{\alpha L_{Yt}^{1-\alpha} A_N^{1-\alpha} x^{\alpha-1}}{\alpha L_{Yt}^{1-\alpha} A_{N-1}^{1-\alpha} x^{\alpha-1}} = \left(\frac{A_N}{A_{N-1}}\right)^{1-\alpha} = (1+\gamma)^{1-\alpha}$$

- Lagging firm $(N-1)$ tries to undercut, but only down to **marginal cost**: $p_{N-1} = r$
- Leading firm (N) can charge $p_N < (1+\gamma)^{1-\alpha} r$ and take the whole market

So, **current market leader** acts like a **monopolist** if innovations are **drastic** enough:

$$\frac{r}{\alpha} < (1+\gamma)^{1-\alpha} r$$

Only $j = N$ is chosen!

Profit maximization for intermediate goods producer

The market-leading intermediate producer's first-order condition is

$$\boldsymbol{r} = p_{jt} + x_{jt}\frac{\partial p_{jt}}{\partial x_{jt}} = p_{jt}\left(1 + \frac{x_{jt}}{p_{jt}}\frac{\partial p_{jt}}{\partial x_{jt}}\right) = p_{jt}\left(1 + (\alpha - 1)\right) = \boldsymbol{\alpha p_{jt}}$$

So, it charges the same **markup** over marginal cost as firms did in Romer (1990):

$$p_{jt} \equiv \boldsymbol{p_t = \frac{1}{\alpha} r}$$

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Intuition: just like before, the **price elasticity of demand** is...

$$\frac{\partial \ln x_{jt}}{\partial \ln p_{jt}} = \frac{p_{jt}}{x_{jt}}\frac{\partial x_{jt}}{\partial p_{jt}} = \boldsymbol{-\frac{1}{1-\alpha}}$$

...so $\alpha \to 1$ looks like perfect competition!

Adding up production

Only the current market leader ($j = N_t$) produces, so they rent **all the capital**:

$$x_{N_t} = K_t.$$

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Substituting this back into the final goods production function...

$$\boldsymbol{Y_t} = L_{Yt}^{1-\alpha} A_{jt}^{1-\alpha} x_{jt}^{\alpha} = L_{Yt}^{1-\alpha} A_{Nt}^{1-\alpha} K_t^{\alpha} = K_t^{\alpha} \left(\boldsymbol{A_{N_t}} L_{Yt} \right)^{1-\alpha}$$

...which is where we started before introducing the microfoundation!

Profits and their growth

How much do final goods producers **spend** on intermediates?

$$
\begin{aligned}
0 = \pi_t^{\text{final}} &= Y_t - wL_{Yt} - p_{N_t}x_{N_t} \\
&= Y_t - \left((1-\alpha)\frac{Y_t}{L_{Yt}}\right)L_{Yt} - p_{N_t}x_{N_t} \\
&= \alpha Y_t - p_{N_t}x_{N_t} \\
p_{N_t}x_{N_t} &= \alpha Y_t
\end{aligned}
$$

How much does the market-leading intermediate producer **earn** in operating profits?

$$\boldsymbol{\pi_{N_t}} = p_{N_t}x_{N_t} - r_t x_{N_t} = (p_{N_t} - \alpha p_{N_t})x_{N_t} = (1-\alpha)p_{N_t}x_{N_t} = \boldsymbol{(1-\alpha)\alpha Y_t}$$

It follows that along a BGP...

$$\boldsymbol{g_\pi} = g_Y = \mathbb{E}[g_A] + g_L = \gamma\mathbb{E}[\dot{N}] + \boldsymbol{g_L}$$

The R&D decision

- Assume intermediate good producers are "small" $\rightarrow$ take $\mathbb{E}[\dot{N}]$ as given
- Pay a **fixed cost** at t in units of labor to create one new idea:

$$F_t = w_t \frac{s_R L_t}{\mathbb{E}[\dot{N}]} = \frac{w_t}{\theta}(s_R L_t)^{1-\lambda} A_t^{1-\phi}$$

- The **value** of a new idea at t is the **PDV of *expected* profits** it could earn:

$$V_t = \mathbb{E}\left[\int_t^T e^{-r\tau}\left(e^{g_\pi \tau}\pi_t\right)\, d\tau\right] = \int_t^\infty e^{-(r+\mathbb{E}[\dot{N}])\tau}\left(e^{g_\pi \tau}\pi_t\right)\, d\tau = \frac{\pi_t}{r - g_\pi + \mathbb{E}[\dot{N}]}$$

  where $\mathbb{E}[\dot{N}]$ effectively "discounts" according to $\mathbb{P}$(**business stealing**)
- A (potential) producer will try to create a new idea if $F_t \leq V_t$ (risk-neutral)
- **Free entry** of potential producers $\rightarrow$ competition ensures $F_t = V_t$ in equilibrium

Putting it all together to pin down s_R on BGP

Start from the free entry condition $F_t = V_t$:

$$\frac{\pi_t}{r - g_\pi + \mathbb{E}[\dot{N}]} = w_t \frac{s_R L_t}{\mathbb{E}[\dot{N}]}$$

Now substitute expressions for π_t, g_π, and w_t from previous slides:

$$\frac{\alpha(1-\alpha)Y_t}{r - \gamma\mathbb{E}[\dot{N}] - g_L + \mathbb{E}[\dot{N}]} = \left[(1-\alpha)\frac{Y_t}{(1-s_R)L_t}\right]\frac{s_R L_t}{\mathbb{E}[\dot{N}]}$$

Rearrange and cancel terms:

$$\frac{s_R}{1-s_R} = \frac{\alpha(1-\alpha)}{1-\alpha}\frac{\mathbb{E}[\dot{N}]}{r - g_L + (1-\gamma)\mathbb{E}[\dot{N}]} \quad \rightarrow \quad s_R = \frac{1}{1 + \frac{r - g_L + (1-\gamma)\mathbb{E}[\dot{N}]}{\alpha\mathbb{E}[\dot{N}]}}$$

(r, $\mathbb{E}[\dot{N}]$ are endogenous but independent of s_R on BGP)

Comparing microfoundations

Expanding varieties (Romer, 1990)

$$g_A^{\text{ss}} = \frac{\boldsymbol{\lambda}}{1-\phi} g_L$$

$$s_R = \frac{1}{1 + \frac{r - g_L}{\alpha g_A^{\text{ss}}}}$$

Quality ladder (Aghion and Howitt, 1992)

$$\mathbb{E}[g_A]^{\text{ss}} = \frac{\boldsymbol{\lambda}}{1-\phi} g_L$$

$$s_R = \frac{1}{1 + \frac{r - g_L + (1-\gamma)\mathbb{E}[\dot{N}]}{\alpha \mathbb{E}[\dot{N}]}} = \frac{1}{1 + \frac{\gamma(r - g_L) + (1-\gamma)\mathbb{E}[g_A]^{\text{ss}}}{\alpha \mathbb{E}[g_A]^{\text{ss}}}}$$

The long-run and transitional dynamics are the same, but **levels** differ w/ s_R:

$$s_R^{\text{EV}} < s_R^{\text{QL}} \iff \mathbb{E}[g_A]^{\text{ss}} < r - g_L$$

How much of US growth (1983–2013) comes from each type of innovation?

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- 27% from **new** varieties of goods (Romer, 1990)

Innovative growth accounting (Klenow and Li, 2021)

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- 27% from **new** varieties of goods (Romer, 1990)
- 13% from **better** varieties of goods *by competing firms* (Aghion and Howitt, 1992)

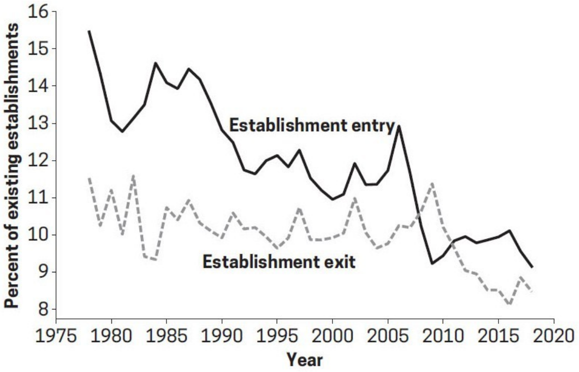
Innovative growth accounting (Klenow and Li, 2021)

How much of US growth (1983–2013) comes from each type of innovation?

- 27% from **new** varieties of goods (Romer, 1990)
- 13% from **better** varieties of goods *by competing firms* (Aghion and Howitt, 1992)
- **60%** from **better** varieties of goods ***by the same firm***
  - Apple iPhone 5 vs. 17 Pro
  - **the Schumpeterian model had no motive for a firm to do this**...
    - "Arrow replacement effect": $V_N - V_{N-1} < V_N = F_t$
    - **drastic** innovation $\longrightarrow$ laggards exit entirely
  - ...**but richer models do** (Aghion et al., 2001; Aghion, Akcigit, and Howitt, 2014)
    - firm improves its own product to **escape competition** and charge a **higher markup**
    - must think about more complex strategic interactions by firms $\longrightarrow$ **not in this class**

The decline in business dynamism. . . (Decker et al., 2016, 2017)

Figure 6.2 Business Dynamism 1978–2018, United States



SOURCE: U.S. Census Business Dynamics Statistics.

NOTE: Establishments refer to individual locations (e.g., plants or stores) that may be part of larger firms. The entry rate is the number of new establishments (meaning they go from zero to positive employees) divided by existing establishments. The exit rate is closing establishments (from positive to zero employees) divided by existing establishments.

. . . what's behind it?

In the Schumpeterian model with **drastic** innovations. . .

$$\textit{entry}\ \text{rate} = \textit{exit}\ \text{rate} = \mathbb{E}[\dot{\boldsymbol{N}}]$$

. . . and, recall, $\boldsymbol{s_R}$ should **decrease** if $\mathbb{E}[\dot{\boldsymbol{N}}]$ declines:

$$\frac{\boldsymbol{s_R}}{1-\boldsymbol{s_R}} = \frac{\alpha(1-\alpha)}{1-\alpha}\frac{\mathbb{E}[\dot{N}]}{r-g_L+(1-\gamma)\mathbb{E}[\dot{N}]}$$

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But we saw in Week 6 that $\boldsymbol{s_R}$ has been **increasing**! **We have a puzzle**. . .

- maybe profit shares, $\alpha(1-\alpha)$, have actually increased (Autor et al., 2020)
- maybe IP protection has changed (Akcigit and Ates, 2021, 2023)

Summary

- focus on ideas $\rightarrow$ a **special kind** of economic good
  - "instructions for mixing raw materials together"
  - nonrival $\rightarrow$ IRS $\rightarrow$ required dropping **neoclassical assumptions**
- build (semi-)endogenous growth models w/ **costly**, **deliberate** idea accumulation
  - share aggregate dynamics in which A grows endogenously...
  - ...b/c firms invent **new** varieties of goods (Romer, 1990) [Week 6]
  - ...b/c firms invent **better** varieties of goods (Aghion and Howitt, 1992) [Week 7]
- but these were just models of "frontier" economies...
  - discuss how technology diffuses to the rest of the world [next time]

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