ECON 164: Theory of Economic Growth

Week 8A: Technology Adoption and Trade

Levi Crews

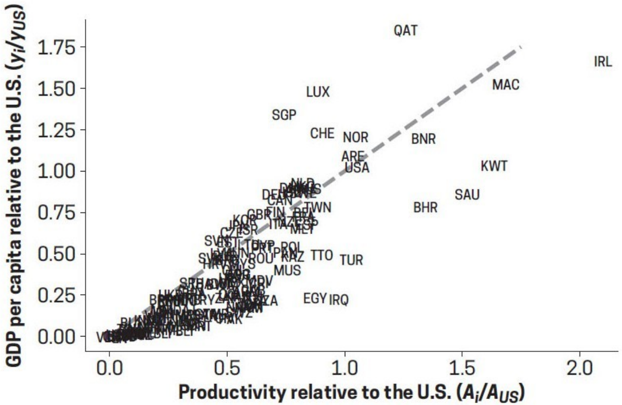
Winter 2026

Last two weeks

- focused on **ideas** → a **special kind** of economic good
  - "instructions for mixing raw materials together"
  - nonrival → IRS → required dropping **neoclassical assumptions**
- built (semi-)endogenous growth models w/ **costly**, **deliberate** idea accumulation
  - share aggregate dynamics in which A grows endogenously...
  - ...b/c firms invent **new** varieties of goods (Romer, 1990)
  - ...b/c firms invent **better** varieties of goods (Aghion and Howitt, 1992)
- but these were just models of "frontier" economies...
  - **today: discuss how technology diffuses to the rest of the world**

Recall development accounting (w/ human capital) (Hall and Jones, 1999)

Figure 7.1 Productivity and GDP per Capita



SOURCE: Authors' calculations from Penn World Tables v10.0 (Feenstra, Inklaar, and Timmer, 2015).

NOTE: Productivity is calculated according to equation (7.6) and the assumptions outlined in the notes of Table 7.1.

How to close technology gaps?

- even w/ human capital, the best predictor of y_i / y_{US} is still A_i / A_{US}
- **why are some countries so far behind? how might they catch up?**

(will always then have $D_t + M_t \leq A_t$)

How to close technology gaps?

- even w/ human capital, the best predictor of y_i / y_{US} is still A_i / A_{US}
- **why are some countries so far behind? how might they catch up?**
- through the lens of the **Romer (1990)** model...

  1. a country can **adopt ideas** D_t from the frontier A_t
  2. a country can **import intermediate goods** M_t from the frontier A_t

(will always then have $D_t + M_t \leq A_t$)

Final and intermediate good producers

Assume a large number of identical firms using the same Cobb-Douglas technology

$$Y_t = (hL_t)^{1-\alpha} \int_0^{D_t} x_{jt}^{\alpha} \, dj$$

where h is the (constant) level of human capital and D_t measures **domestic varieties**

Final and intermediate good producers

Assume a large number of identical firms using the same Cobb-Douglas technology

$$Y_t = (hL_t)^{1-\alpha} \int_0^{\boldsymbol{D_t}} x_{jt}^{\alpha} \, dj$$

where h is the (constant) level of human capital and $\boldsymbol{D_t}$ measures **domestic varieties**

As in **Romer (1990)**, varieties are symmetric and only require capital to produce:

$$\int_0^{\boldsymbol{D_t}} x_{jt} \, dj = K_t \qquad \rightarrow \qquad x_{jt} = x_t = \frac{K_t}{\boldsymbol{D_t}}$$

Production, capital, and labor

The aggregate production function is then (with Harrod-neutral productivity)

$$Y_t = K_t^{\alpha}(\boldsymbol{D_t} h L_t)^{1-\alpha}$$

Production, capital, and labor

The aggregate production function is then (with Harrod-neutral productivity)

$$Y_t = K_t^{\alpha}(\boldsymbol{D_t} h L_t)^{1-\alpha}$$

Capital and labor accumulation as in the **Solow** model:

$$\dot{K}_t = s_I Y_t - \delta K_t$$
$$\dot{L}_t = g_L L_t$$

Production, capital, and labor

The aggregate production function is then (with Harrod-neutral productivity)

$$Y_t = K_t^{\alpha}(\boldsymbol{D_t} h L_t)^{1-\alpha}$$

Capital and labor accumulation as in the **Solow** model:

$$\dot{K}_t = s_I Y_t - \delta K_t$$
$$\dot{L}_t = g_L L_t$$

$\longrightarrow$ must be that, in the long run, $\boldsymbol{g_y^{\text{ss}} = g_D}$

Not R&D, but technology adoption

For some parameter $\zeta \in [0, 1]$, the adoption of ideas obeys

$$\dot{D}_t = \psi h A_t^{\zeta} D_t^{1-\zeta}$$

Not R&D, but technology adoption

For some parameter $\zeta \in [0,1]$, the adoption of ideas obeys

$$\dot{D}_t = \psi h A_t^{\zeta} D_t^{1-\zeta}$$

- human capital h affects the **rate** of adoption
    - need skilled labor to understand and implement frontier ideas
    - could explicitly model share s_D of skilled labor engaged in adoption $\rightarrow$ **not today** (buying licensing rights, purchasing patents, imitating products, etc.)

Not R&D, but technology adoption

For some parameter $\zeta \in [0, 1]$, the adoption of ideas obeys

$$\dot{D}_t = \psi h A_t^{\zeta} D_t^{1-\zeta}$$

- human capital h affects the **rate** of adoption
  - need skilled labor to understand and implement frontier ideas
  - could explicitly model share s_D of skilled labor engaged in adoption $\rightarrow$ **not today** (buying licensing rights, purchasing patents, imitating products, etc.)
- frontier ideas A_t are treated as **exogenous** by adopting country

Not R&D, but technology adoption

For some parameter $\zeta \in [0,1]$, the adoption of ideas obeys

$$\dot{D}_t = \psi h A_t^{\zeta} D_t^{1-\zeta}$$

- human capital h affects the **rate** of adoption
  - need skilled labor to understand and implement frontier ideas
  - could explicitly model share s_D of skilled labor engaged in adoption → **not today** (buying licensing rights, purchasing patents, imitating products, etc.)
- frontier ideas A_t are treated as **exogenous** by adopting country
- parameter ζ governs relative importance of A_t vs. D_t in ability to adopt
  - $\zeta \to 1$: matters more to have more ideas to borrow/copy/purchase from frontier
  - $\zeta \to 0$: matters more to have better domestic technology already (e.g., Internet)
  - *note*: could have been more general than Cobb-Douglas

Growth of productivity

Divide both sides through by D_t to get. . .

$$g_D \equiv \frac{\dot{D}_t}{D_t} = \psi h \left(\frac{A_t}{D_t} \right)^{\zeta}$$

Growth of productivity

Divide both sides through by D_t to get...

$$g_D \equiv \frac{\dot{D}_t}{D_t} = \psi h \left(\frac{A_t}{D_t}\right)^{\zeta}$$

A **steady state** will be where g_D is constant $\longrightarrow \frac{A_t}{D_t}$ is constant

how do we find it?

Growth of productivity

Divide both sides through by D_t to get...

$$g_D \equiv \frac{\dot{D}_t}{D_t} = \psi h \left(\frac{A_t}{D_t}\right)^{\zeta}$$

A **steady state** will be where g_D is constant $\rightarrow \frac{A_t}{D_t}$ is constant

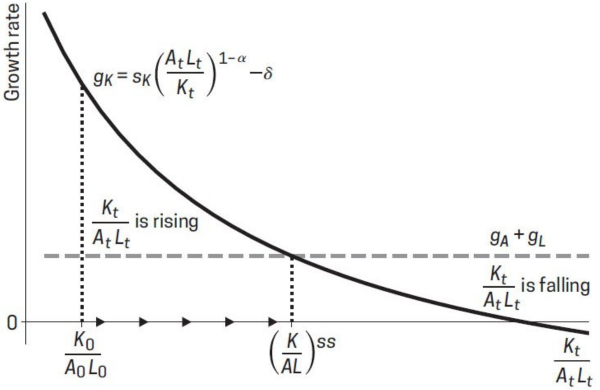
how do we find it?

Recall **similar expressions** from the Solow ($\boldsymbol{g_K}$) and Romer ($\boldsymbol{g_A}$) models...

$$g_K = s_I \left(\frac{A_t L_t}{K_t}\right)^{1-\alpha} - \delta \qquad g_A = \theta s_R^{\lambda} \left(\frac{L_t^{\lambda}}{A_t^{1-\phi}}\right)$$

Use a phase diagram

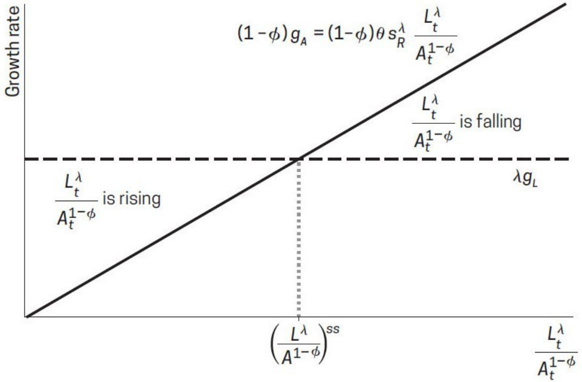
Figure 2.2 The Dynamics of the K_t/A_tL_t Ratio



NOTE: The dark line plots the growth rate of capital, g_K, from equation (2.11) against the K_t/A_tL_t ratio. The growth rate of AL, which is $g_A + g_L$, is plotted in the dashed line. Everywhere the dashed line is above the solid line, the K_t/A_tL_t ratio must be falling, as $g_K < g_A + g_L$. Everywhere the dashed line is below the solid line, the K_t/A_tL_t ratio must be rising, as $g_K > g_A + g_L$.

Use a phase diagram

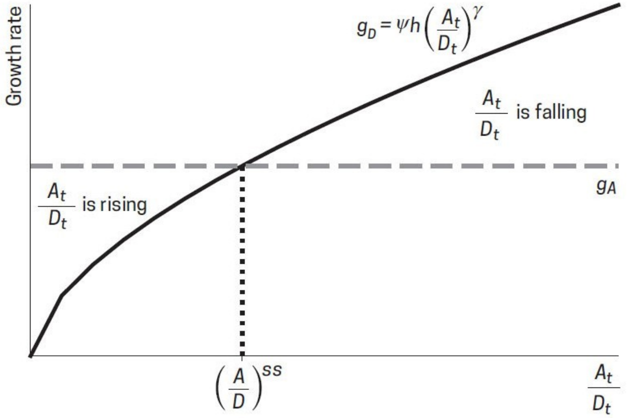
Figure 5.1 The Dynamics of Productivity Growth



NOTE: The two curves plot the growth rate of the numerator and denominator of the ratio $L_t^\lambda / A_t^{1-\phi}$. The growth rate of the numerator is λg_L, plotted as the horizontal line. The growth rate of the denominator is $(1-\phi)g_A$, which from equation (5.8) is increasing in the ratio. The economy reaches a steady state where the two lines intersect, and at that point the growth rate of productivity is $g_A^{ss} = \frac{\lambda}{1-\phi} g_L$.

Use a phase diagram

Figure 7.2 The Dynamics of Domestic Technology



NOTE: The growth rate of domestic technology, D_t, is from equation (7.10) and is rising with the ratio A_t/D_t. The growth rate of frontier technology, g_A, is taken as given in the domestic economy and does not change with A_t/D_t.

Characterizing the steady state

The steady-state growth rate of **domestic productivity** is...

$$g_D^{\text{ss}} = g_A$$

...and the steady-state ratio is

$$\left(\frac{A}{D}\right)^{\text{ss}} = \left(\frac{g_A}{\psi h}\right)^{\frac{1}{\zeta}},$$

which means, along the BGP,

$$D_t^{\text{BGP}} = \left(\frac{\psi h}{g_A}\right)^{\frac{1}{\zeta}} A_t^{\text{BGP}}$$

Let $\tilde{k}_t \equiv K_t / \left(D_t h L_t\right) \to$ *notice* h*!*

Output per capita along the BGP is then

$$y_t^{\text{BGP}} = \left(\tilde{k}^{\text{ss}}\right)^{\alpha} h D_t^{\text{BGP}}$$

$$= \left(\tilde{k}^{\text{ss}}\right)^{\alpha} h \left(\frac{\psi h}{g_A}\right)^{\frac{1}{\zeta}} A_t^{\text{BGP}}$$

where, as in the Solow model,

$$\tilde{k}^{\text{ss}} = \left(\frac{s_I}{g_A + g_L + \delta}\right)^{\frac{1}{1-\alpha}},$$

The double role of human capital

The level of human capital enters twice:

$$y_t^{\mathsf{BGP}} = \left(\tilde{k}^{\mathsf{ss}}\right)^{\alpha} \boldsymbol{h} D_t^{\mathsf{BGP}}$$

- $\boldsymbol{h}$ determines the skill of workers in final good production

The double role of human capital

The level of human capital enters twice:

$$y_t^{\mathsf{BGP}} = \left(\tilde{k}^{\mathsf{ss}}\right)^{\alpha} \boldsymbol{h} D_t^{\mathsf{BGP}} = \left(\tilde{k}^{\mathsf{ss}}\right)^{\alpha} h \left(\frac{\psi \boldsymbol{h}}{g_A}\right)^{\frac{1}{\zeta}} A_t^{\mathsf{BGP}}$$

- $\boldsymbol{h}$ determines the skill of workers in final good production
- $\boldsymbol{h}^{\frac{1}{\zeta}}$ determines how much technology adoption happens

The double role of human capital

The level of human capital enters twice:

$$y_t^{\mathsf{BGP}} = \left(\tilde{k}^{\mathsf{ss}}\right)^{\alpha} \boldsymbol{h} D_t^{\mathsf{BGP}} = \left(\tilde{k}^{\mathsf{ss}}\right)^{\alpha} h \left(\frac{\psi \boldsymbol{h}}{g_A}\right)^{\frac{1}{\zeta}} A_t^{\mathsf{BGP}}$$

- $\boldsymbol{h}$ determines the skill of workers in final good production
- $\boldsymbol{h}^{\frac{1}{\zeta}}$ determines how much technology adoption happens

Recall our discussion of development accounting w/ human capital (Week 4 Slides 29–38)

- maybe we were **understating** the role of $\boldsymbol{h}$ and **overstating** the role of $\boldsymbol{A}$

The double role of human capital

The level of human capital enters twice:

$$y_t^{\text{BGP}} = \left(\tilde{k}^{\text{ss}}\right)^{\alpha} \boldsymbol{h} D_t^{\text{BGP}} = \left(\tilde{k}^{\text{ss}}\right)^{\alpha} h \left(\frac{\psi \boldsymbol{h}}{g_A}\right)^{\frac{1}{\zeta}} A_t^{\text{BGP}}$$

- $\boldsymbol{h}$ determines the skill of workers in final good production
- $\boldsymbol{h}^{\frac{1}{\zeta}}$ determines how much technology adoption happens

Recall our discussion of development accounting w/ human capital (Week 4 Slides 29–38)

- maybe we were **understating** the role of $\boldsymbol{h}$ and **overstating** the role of $\boldsymbol{A}$
- **Hendricks and Schoellman (2018)**: check Δw_i for migrants $\rightarrow$ $\boldsymbol{h}$ matters most

Adding imports

Assume a large number of identical firms using the same Cobb-Douglas technology

$$Y_t = (hL_t)^{1-\alpha} \int_0^{\boldsymbol{D_t}+\boldsymbol{M_t}} x_{jt}^{\alpha} \, dj$$

where $\boldsymbol{D_t}$ measures **domestic varieties** and $\boldsymbol{M_t}$ measures **imported varieties**

Adding imports

Assume a large number of identical firms using the same Cobb-Douglas technology

$$Y_t = (hL_t)^{1-\alpha} \int_0^{D_t + M_t} x_{jt}^{\alpha} \, dj$$

where D_t measures **domestic varieties** and M_t measures **imported varieties**

Again as in Romer (1990), varieties are symmetric and only require capital to produce, but now there's **trade**: let x_{jt} still be quantity **demanded** of $j \in [0, D_t + M_t]$...

$$\int_0^{D_t + M_t} x_{jt} \, dj = K_t \qquad \rightarrow \qquad x_{jt} = x_t = \frac{K_t}{D_t + M_t}$$

...but now z_{jt} is quantity **supplied** of $j \in [0, D_t]$

$$\int_0^{D_t} z_{jt} \, dj = K_t \qquad \rightarrow \qquad \underbrace{D_t(z_t - x_t)}_{\text{exports}} = \underbrace{M_t x_t}_{\text{imports}}$$

Production, capital, and labor

The aggregate production function is then (with Harrod-neutral productivity)

$$Y_t = K_t^{\alpha}[(\boldsymbol{D_t} + \boldsymbol{M_t})hL_t]^{1-\alpha}$$

Production, capital, and labor

The aggregate production function is then (with Harrod-neutral productivity)

$$Y_t = K_t^\alpha[(\boldsymbol{D_t} + \boldsymbol{M_t})hL_t]^{1-\alpha} = K_t^\alpha(\boldsymbol{D_t}hL_t)^{1-\alpha}\left(1 + \frac{\boldsymbol{M_t}}{\boldsymbol{D_t}}\right)^{1-\alpha}$$

Capital and labor accumulation as in the **Solow** model:

$$\dot{K}_t = s_I Y_t - \delta K_t$$
$$\dot{L}_t = g_L L_t$$

Production, capital, and labor

The aggregate production function is then (with Harrod-neutral productivity)

$$Y_t = K_t^\alpha [(\boldsymbol{D_t} + \boldsymbol{M_t}) h L_t]^{1-\alpha} = K_t^\alpha (\boldsymbol{D_t} h L_t)^{1-\alpha} \left(1 + \frac{\boldsymbol{M_t}}{\boldsymbol{D_t}}\right)^{1-\alpha}$$

Capital and labor accumulation as in the **Solow** model:

$$\dot{K}_t = s_I Y_t - \delta K_t$$
$$\dot{L}_t = g_L L_t$$

$\longrightarrow$ still must be that, in the long run, $\boldsymbol{g_y^{\text{ss}} = g_D}$

Characterizing the BGP

- same adoption rule $\dot{D}_t \rightarrow$ same phase diagram to find the steady state

Characterizing the BGP

- same adoption rule $\dot{\boldsymbol{D}}_t \rightarrow$ same phase diagram to find the steady state
- only change is additional scaling term from **imports**:

$$y_t^{\text{BGP}} = \left(\tilde{k}^{\text{ss}}\right)^{\alpha} h \left(\frac{\psi h}{g_A}\right)^{\frac{1}{\zeta}} \left[1 + \left(\frac{\boldsymbol{M}}{\boldsymbol{D}}\right)^{\text{ss}}\right]^{1-\alpha} A_t^{\text{BGP}}$$

Characterizing the BGP

- same adoption rule $\dot{\boldsymbol{D}}_t \to$ same phase diagram to find the steady state
- only change is additional scaling term from **imports**:

$$y_t^{\text{BGP}} = \left(\tilde{k}^{\text{ss}}\right)^{\alpha} h \left(\frac{\psi h}{g_A}\right)^{\frac{1}{\zeta}} \left[1 + \left(\frac{\boldsymbol{M}}{\boldsymbol{D}}\right)^{\text{ss}}\right]^{1-\alpha} A_t^{\text{BGP}}$$

- didn't specify how to pin down $\left(\frac{\boldsymbol{M}}{\boldsymbol{D}}\right)^{\text{ss}} \to$ just assume it's constant for this class
  - implicitly always assuming $\boldsymbol{M}_t \leq A_t - \boldsymbol{D}_t$, though $\to 1 + \left(\frac{\boldsymbol{M}}{\boldsymbol{D}}\right)^{\text{ss}} \leq \left(\frac{A}{\boldsymbol{D}}\right)^{\text{ss}}$

Characterizing the BGP

- same adoption rule $\dot{D}_t \rightarrow$ same phase diagram to find the steady state
- only change is additional scaling term from **imports**:

$$y_t^{\text{BGP}} = \left(\tilde{k}^{\text{ss}}\right)^{\alpha} h \left(\frac{\psi h}{g_A}\right)^{\frac{1}{\zeta}} \left[1 + \left(\frac{M}{D}\right)^{\text{ss}}\right]^{1-\alpha} A_t^{\text{BGP}}$$

- didn't specify how to pin down $\left(\frac{M}{D}\right)^{\text{ss}} \rightarrow$ just assume it's constant for this class
  - implicitly always assuming $M_t \leq A_t - D_t$, though $\rightarrow 1 + \left(\frac{M}{D}\right)^{\text{ss}} \leq \left(\frac{A}{D}\right)^{\text{ss}}$
- there is a positive level effect of **more imports** $\rightarrow$ **why?**
  - diminishing returns to each variety ($\alpha < 1$)...

Characterizing the BGP

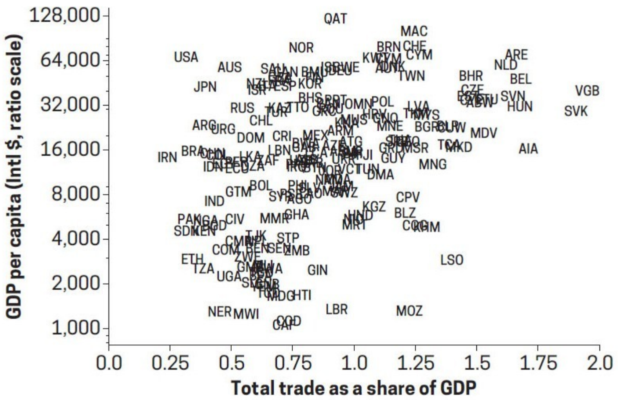
- same adoption rule $\dot{D}_t \rightarrow$ same phase diagram to find the steady state
- only change is additional scaling term from **imports**:

$$y_t^{\text{BGP}} = \left(\tilde{k}^{\text{ss}}\right)^{\alpha} h \left(\frac{\psi h}{g_A}\right)^{\frac{1}{\zeta}} \left[1 + \left(\frac{M}{D}\right)^{\text{ss}}\right]^{1-\alpha} A_t^{\text{BGP}}$$

- didn't specify how to pin down $\left(\frac{M}{D}\right)^{\text{ss}}$ $\rightarrow$ just assume it's constant for this class
  - implicitly always assuming $M_t \leq A_t - D_t$, though $\rightarrow 1 + \left(\frac{M}{D}\right)^{\text{ss}} \leq \left(\frac{A}{D}\right)^{\text{ss}}$
- there is a positive level effect of **more imports** $\rightarrow$ **why?**
  - diminishing returns to each variety ($\alpha < 1$)...
  - ...means it's better to have a small amount of more varieties $\rightarrow$ **motive for trade**

Trade openness is correlated with higher levels of GDP per capita

Figure 7.3 Total Trade and GDP per Capita, 2019



SOURCE: Authors' calculations from Penn World Tables v10.0 (Feenstra, Inklaar, and Timmer, 2015).

NOTE: Total trade is the sum of exports and imports, divided by GDP, all in current national prices. GDP per capita is the purchasing power parity (PPP)-adjusted level.

$$\frac{\text{Imports}}{\text{GDP}} = \frac{M_t x_t}{Y_t} = \frac{M_t}{D_t + M_t}\frac{K_t}{Y_t}$$

- no countries in **lower right** → **closed** are poor / poor don't **trade**
- countries in **top left** tend to be at the tech. frontier → **invent** the varieties, don't **import** as many

Summary

- neoclassical growth model: always assumed common g_A across countries

Summary

- neoclassical growth model: always assumed common g_A across countries
- **today**: one way to justify that assumption *in the (very) long run*
  - a few frontier countries invent new ideas ($\dot{A}$) using people (g_L) ...
  - ... **while the rest adopt ($\dot{D}$) or import them (M)**
  - w/ production and physical capital accumulation as in the NGM

Summary

- neoclassical growth model: always assumed common g_A across countries
- today: one way to justify that assumption *in the (very) long run*
  - a few frontier countries invent new ideas ($\dot{A}$) using people (g_L) ...
  - ... **while the rest adopt ($\dot{D}$) or import them (M)**
  - w/ production and physical capital accumulation as in the NGM
- but gaps—in levels *and* growth rates—can persist for a long time...
  - transitional dynamics from **physical capital** as in NGM
  - transitional dynamics from **technology** through adoption and trade

References

Aghion, Philippe and Peter Howitt. 1992. "A model of growth through creative destruction." *Econometrica* 60 (2):323–351.

Hall, Robert E. and Charles I. Jones. 1999. "Why do some countries produce so much more output per worker than others?" *Quarterly Journal of Economics* 114 (1):83–116.

Hendricks, Lutz and Todd Schoellman. 2018. "Human capital and development accounting: New evidence from wage gains at migration." *Quarterly Journal of Economics* 133 (2):665–700.

Romer, Paul M. 1990. "Endogenous technological change." *Journal of Political Economy* 98 (5, Part 2):S71–S102.