ECON 164: Theory of Economic Growth

Week 8B: Population and Fertility

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Winter 2026

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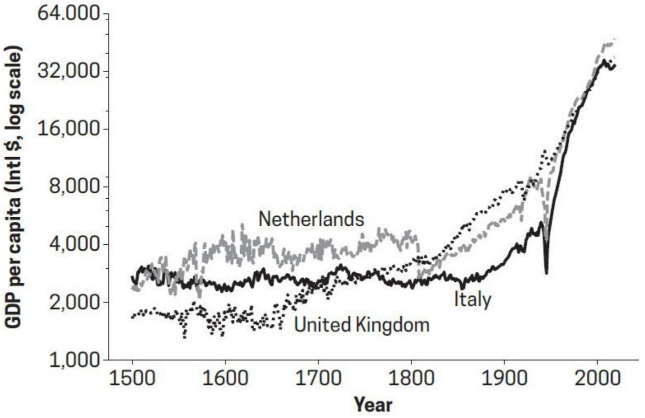
- **why**: ideas require people + ideas aren't diluted by more people (*nonrival*)
- but this is in stark contrast with once-dominant hypothesis of **Malthus**: "misery is an absolute necessary consequence" of population growth
- **this lecture**: two new ingredients
  1. endogenous pop. growth tied to income levels (*is there enough food?*)
  2. a resource in fixed supply (agricultural land)
- use this to think through the **transition** from Malthusian to modern growth

Let's take Fact 3 from Week 1...

Figure 1.3 GDP per Capita, 1500–2019, Selected Countries



SOURCE: Author's calculations from Broadberry et al. (2015), Malanima (2011), van Zanden and van Leeuwen (2012), and Bolt and van Zanden (2020).

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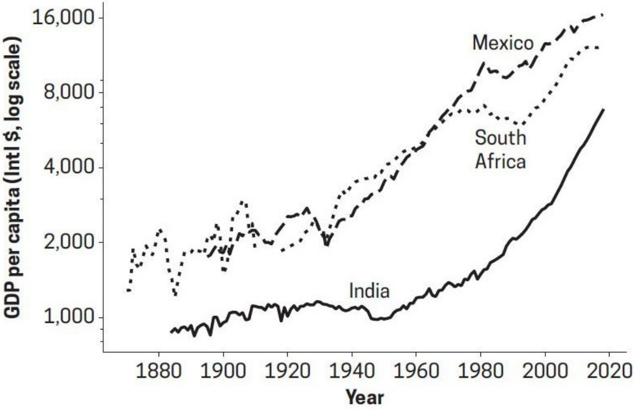
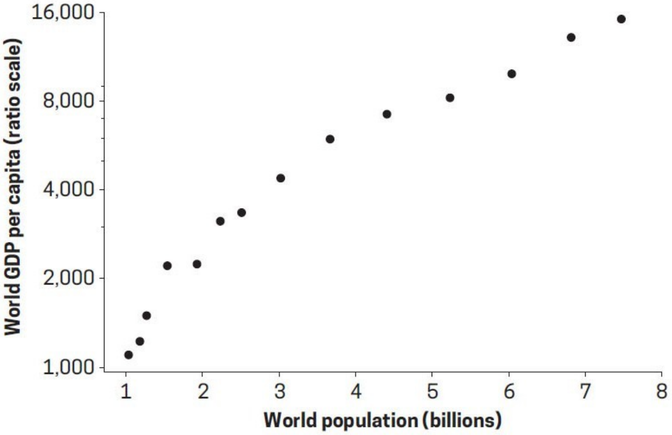
Figure 1.4 GDP per Capita, 1870–2019, Selected Countries



SOURCE: Author's calculations from Barro and Ursua (2008), Broadberry, Custodis, and Gupta (2015), Fourie and Zanden (2013), and Bolt and van Zanden (2020).

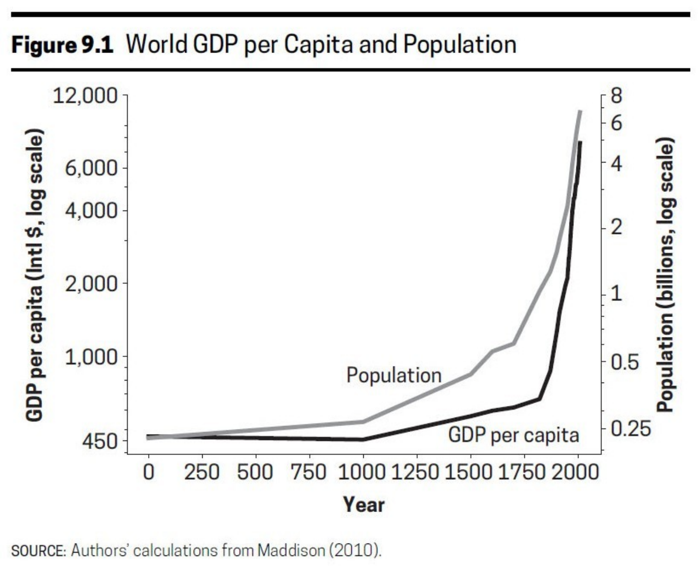
And slide 25 from Week 6...

Figure 4.6 World Population and GDP per Capita, 1820–2019



SOURCE: Authors' calculations from Bolt and van Zanden (2020).

. . . and extend them back to 0 CE



Three eras of economic growth (Galor and Weil, 2000)

1. **Malthusian epoch** (pre-1800)
   - low levels and growth rates of worldwide GDP per capita (0–0.14%)...
   - ...and of global population (0.02–0.27%)
   - no human capital investment
2. **Post-Malthusian epoch** (1800–1940)
   - accelerating growth rates of worldwide GDP per capita (0.5% $\rightarrow$ 1.3%)...
   - ...and of global population (0.4% $\rightarrow$ 1.9%)
   - but still little human capital investment
3. **Modern growth epoch** (post-1940)
   - steady growth of the frontier (w/ hints of a slowdown)...
   - ...despite *declining* population growth rates at higher levels of GDP per capita
   - accelerating human capital investment

The Malthusian economy

Suppose production is Cobb-Douglas in effective labor and a **fixed supply** of land ($\boldsymbol{X}$):

$$Y_t = \boldsymbol{X}^{\beta} \left(\boldsymbol{B_t} L_t\right)^{1-\beta}$$

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$$\boldsymbol{g_y} = \beta(g_A - g_X - g_L) = \boldsymbol{-\beta g_L}$$

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Why? Unlike physical capital in NGM, **can't accumulate land $\boldsymbol{X}$!**

Endogenizing population growth

Think of y_t as **food per capita** $\rightarrow$ need some **subsistence level $\bar{c}$** to survive

Suppose, then, that population growth obeys

$$g_L = \nu(y_t - \bar{\boldsymbol{c}})$$

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Suppose, then, that population growth obeys

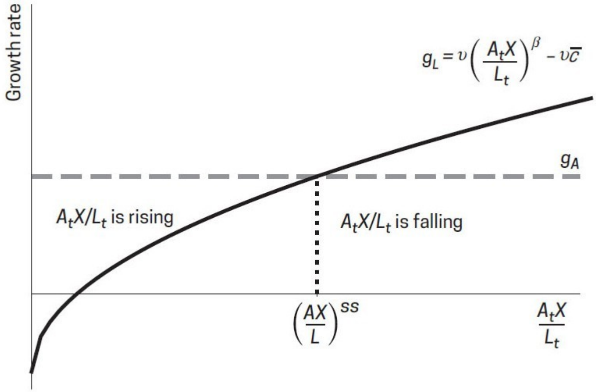
$$g_L = \nu(y_t - \bar{\boldsymbol{c}}) = \nu \left(\frac{A_t \boldsymbol{X}}{L_t} \right)^{\beta} - \nu \bar{\boldsymbol{c}} \lesseqgtr 0$$

A **steady state** will be where g_L is constant $\rightarrow$ $\frac{A_t \boldsymbol{X}}{L_t}$ is constant

how do we find it?

Use a phase diagram

Figure 9.2 The Dynamics of the Malthusian Model



NOTE: The black line plots the growth rate g_L against the ratio A_tX/L_t, as in equation (9.2). The gray line plots the growth rate of productivity, g_A, which is constant regardless of the ratio. Note that the ratio A_tX/L_t plotted on the x-axis determines GDP per capita in equation (9.1). The figure also assumes that $g_X = 0$, or that the stock of land is constant.

Characterizing the steady state

The steady-state growth rate of **population** is...

$$g_L^{\text{ss}} = g_A$$

...and the steady-state ratio is

$$\left(\frac{A\boldsymbol{X}}{L}\right)^{\text{ss}} = \left(\frac{g_A}{\nu} + \bar{\boldsymbol{c}}\right)^{\frac{1}{\beta}},$$

which means, along the BGP,

$$L_t^{\text{BGP}} = \left(\frac{\nu}{g_A + \nu\bar{\boldsymbol{c}}}\right)^{\frac{1}{\beta}} \boldsymbol{X} A_t^{\text{BGP}}$$

Output per capita along the BGP is then

$$y_t^{\text{BGP}} = \left[\left(\frac{A\boldsymbol{X}}{L}\right)^{\text{ss}}\right]^{\beta} = \frac{g_A}{\nu} + \bar{\boldsymbol{c}} \equiv y^{\text{ss}}$$

...and growth in GDP per capita is

$$\begin{aligned} \boldsymbol{g_y^{\text{ss}}} &= \beta(g_A - g_L^{\text{ss}}) \\ &= \beta(g_A - g_A) = \mathbf{0} \end{aligned}$$

Why? Economy eats away its own prosperity!

Tracing the epochs (so far)

Two simple ingredients can generate a **Malthusian economy**:

1. limited supply of land $\boldsymbol{X}$
2. positive relationship between income and population growth

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But to explain the **transition to modern growth**, we'll need to introduce two more:

3. **endogenous** technological change driven by population growth
4. a **reversal** of #2 at high levels of income

If population (growth) determines g_A... (Kremer, 1993)

Think about the (semi-)endogenous growth model we developed the last few weeks...

$$g_A = \theta \frac{(s_R L_t)^\lambda}{A_t^{1-\phi}}$$

...but suppose we're **way below** steady-state $\frac{L^\lambda}{A^{1-\phi}} \rightarrow \boldsymbol{g_A}$ will be **rising**

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We can see this more clearly in the (fully) endogenous case w/ $\lambda = \phi = s_R = 1$:

$$\boldsymbol{g_A = \theta L_t}$$

. . . then the g_A line keeps shifting up

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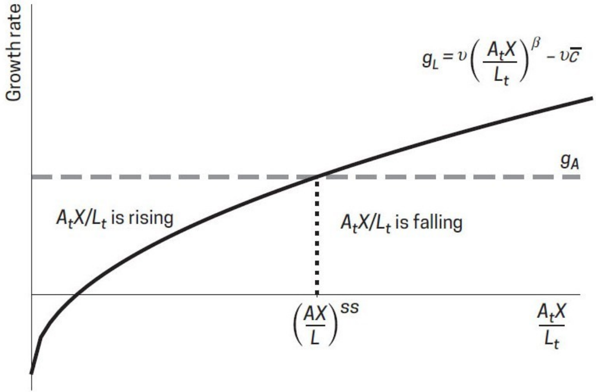

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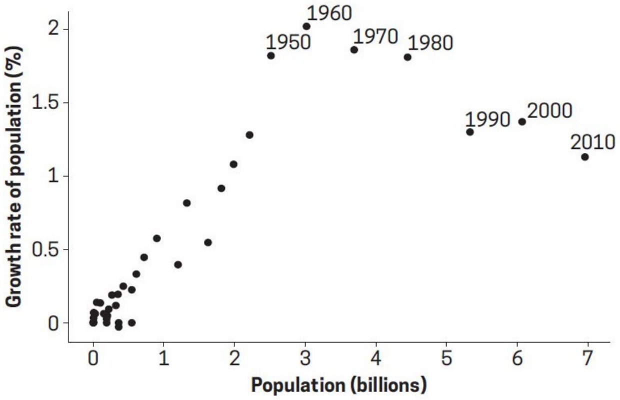
This matches all of history until the 1970s. . .

Figure 9.3 Population Growth and Population Size, 1 Million B.C.E. to 2020



SOURCE: Authors' calculations from Kremer (1993) and U.S. Census Bureau data on world population.

. . . but what about after that?

- The Malthusian model by itself couldn't generate that figure up thru 1960s → needed positive effect of population on technological progress to dominate
- But now we need to explain. . .
  - the drop in population growth in the 1970s and after
  - lack of exploding growth predicted by Kremer (1993)
- . . . so, must be some mechanism that eventually tempers feedback b/w g_A and g_L

The family decision problem

Suppose each identical family **consumes** ($\boldsymbol{c}$) and **has children** ($\boldsymbol{n}$) to solve

$$\max_{c,n} U = n^{\nu} c^{1-\nu} \text{ s.t. } y = c + p_n n \quad [\lambda]$$

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Take first-order conditions:

$$\lambda p_n = \nu \frac{U}{n}, \qquad \lambda = (1-\nu)\frac{U}{c}$$

and so

$$p_n = \left(\frac{\nu}{1-\nu}\right)\frac{c}{n} \quad \rightarrow \quad \boldsymbol{n = \frac{\nu y}{p_n}}$$

The price of children

Suppose that the price of children evolves according to...

$$p_n = \bar{c}e^{\eta y}$$

...which we can rationalize in a few ways:

- rising **opportunity cost** of child rearing as income grows
- rising **relative price** of quantity vs. quality of childhood
- rising **cost of education** as the frontier of knowledge advances

The population growth function

With all families identical, we can combine the last two slides to get...

$$n = \frac{\nu y}{\bar{c} e^{\eta y}} \quad \rightarrow \quad g_L = \frac{\nu y}{\bar{c} e^{\eta y}}$$

The population growth function

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$$\boldsymbol{n = \frac{\nu y}{\bar{c}e^{\eta y}}} \quad \rightarrow \quad \boldsymbol{g_L = \frac{\nu y}{\bar{c}e^{\eta y}}}$$

...which, for low y, behaves like our Malthusian model, $g_L = \nu(y - \bar{c})$, but generally

$$\frac{\partial g_L}{\partial y} = \frac{\nu\eta}{\bar{c}e^{\eta y}}\left(\frac{1}{\eta} - y\right)$$

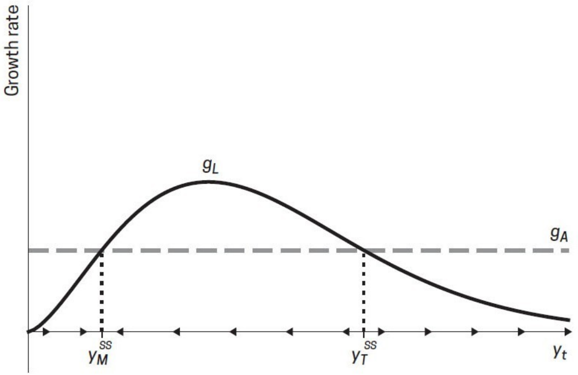
whose sign depends on $\boldsymbol{y}$ vs. $\boldsymbol{1/\eta}$

$$\frac{\partial g_L}{\partial y}\begin{cases} > 0 & \text{if } y < \frac{1}{\eta} \\ = 0 & \text{if } y = \frac{1}{\eta} \\ < 0 & \text{if } y > \frac{1}{\eta} \end{cases}$$

and whose limit is $\boldsymbol{\lim_{y\to\infty} \frac{\partial g_L}{\partial y} = 0}$, so eventually **$g_L$ becomes constant**

The Malthusian model with realistic population growth

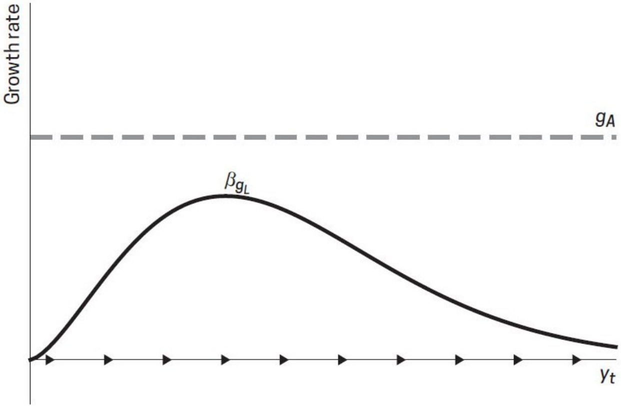
Figure 9.4 The Dynamics of the Malthusian Model with Realistic Population Growth



NOTE: The black line plots the growth rate of population against GDP per capita, y_t, which from equation (9.1) is equal to the ratio $(A_t X/L_t)^\beta$. This function allows for falling g_L at some level. The gray line is the growth rate of productivity. This creates two intersections and two steady states. y_M^{ss} is stable, while y_T^{ss} is unstable.

Now reintroduce technological progress

Figure 9.5 The Transition to Sustained Growth



NOTE: The black line plots the growth rate of population against y_t, allowing for falling g_L at some level. The gray line is the growth rate of productivity. In this setting $g_A > g_L$ at every point and sustained growth in living standards follows.

Characterizing the steady state

Assuming the population growth settles down to some rate g_L^{ss}, then...

$$g_A^{\mathsf{ss}} = \frac{1-\beta}{\beta}\frac{\lambda}{1-\phi}g_L^{\mathsf{ss}}$$

...and growth in GDP per capita is then

$$g_y^{\mathsf{ss}} = \beta(g_A^{\mathsf{ss}} - g_L^{\mathsf{ss}}) = \left((1-\beta)\frac{\lambda}{1-\phi} - \beta\right)g_L^{\mathsf{ss}}$$

References

Galor, Oded. 2011. *Unified growth theory*. Princeton, NJ: Princeton University Press.

Galor, Oded and David N. Weil. 2000. "Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond." *American Economic Review* 90 (4):806–828.

Kremer, Michael. 1993. "Population growth and technological change: One million BC to 1990." *Quarterly Journal of Economics* 108 (3):681–716.