Optimal transport networks in spatial equilibrium

Fajgelbaum & Schaal (*Econometrica* 2020)

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Transport infrastructure matters — How should we allocate it?

Motivation:

- **in theory:** transport infrastructure, **through trade costs**, shapes spatial distribution of prices, real incomes, and trade flows (Allen-Arkolakis 2014; Donaldson-Hornbeck 2016)
- **in practice:** billions spent on roads, airports, seaports, . . .

Question: **How should we allocate transport infrastructure across space?**

- are observed networks suboptimal?
- if so, how large are the losses?

What we need: a method to **identify the best set** of infrastructure investments

This paper provides such a method and applies it to European roads

Theory: optimal transport in GE

- arbitrary geography, productivity, pop.
- solve planner's global optimization problem over space of networks

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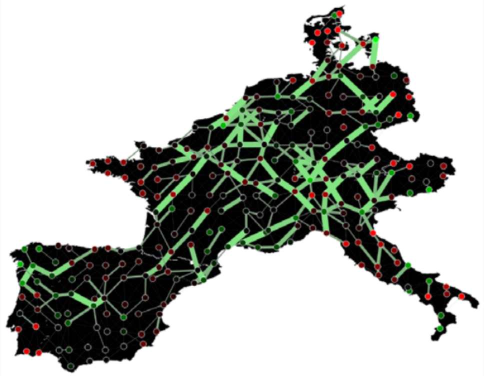
Application: European road network

- welfare gain from optimal 50% expansion ≈ **2.5%**
- added links broadly similar to TEN-T proposal



How the model works

Planner solves

$$\max_{\{\text{Roads}_{jk}\}} \max_{\{\text{Flows}_{jk}\}} \max_{\{\text{Activity}_j\}} \sum_j \text{Welfare}_j$$

subject to

1. local consumption feasibility
2. local factor market clearing
3. road construction feasibility
4. balanced flows
5. non-negativity

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- **optimal allocation sub-problem**: given roads and flows, solve **neoclassical trade model**
  - arbitrary geography, productivity, pop.
  - roads & flows pin down trade costs
  - **has well-known solutions**

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- **optimal flow sub-problem**: given roads, what are optimal trade flows?
  - shut off GE effects $\implies$ well-known problem from optimal transport lit. (see Galichon 2016, Ch8)
  - well-known solution methods: **duality techniques** to derive *potential field*
  - **here:** same methods work in GE in field of **prices**

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- **network design problem**:
  - under assumptions: **whole problem is convex**, can use dual techniques in price space
  - otherwise: use dual techniques to solve sub-problems jointly, then *simulated annealing* to approximate global max

Model

Preferences and production = Neoclassical trade model

Environment:

- discrete set of locations $\mathcal{J} = \{1, \ldots, J\}$
- N traded goods, aggregated by $D_j(\cdot)$
- 1 non-traded good in fixed supply, H_j
- labor L_j (fixed or mobile)

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Preferences: homothetic and concave

$$u_j = U(c_j, h_j)$$

with

$$c_j L_j = D_j(D_j^1, \ldots, D_j^N)$$

where D_j is h.o.d. 1 and concave

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Production:

- fixed factors $\mathbf{V}_j = (V_j^1, \ldots, V_j^M)$
  - immobile across locations
  - mobile across sectors
- intermediates $\mathbf{X}_j = (X_j^1, \ldots, X_j^N)$
- **technology:** neoclassical or constant

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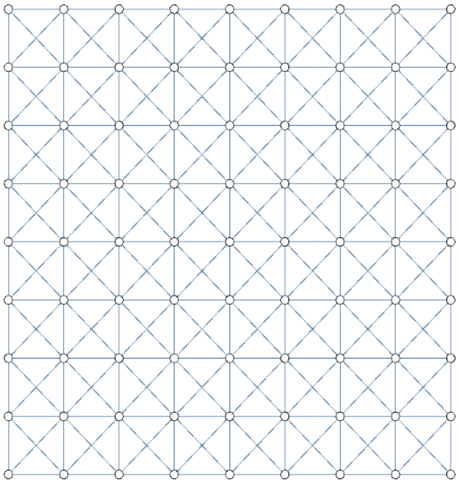
Nests neoclassical trade models:

- Armington
- Ricardian
- factor-endowment

Locations $\mathcal{J}$ are arranged on an undirected graph

- J nodes in set $\mathcal{J}$
- $\mathcal{E} \equiv$ set of edges (unordered pairs of $\mathcal{J}$)
- $\mathcal{N}(j) \equiv$ set of neighbors of location j
- **goods only shipped through connected locations** $k \in \mathcal{N}(j)$
- not necessarily geographically contiguous: airports, seaports



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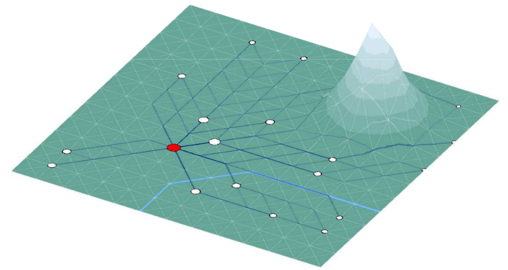
- **can handle topography, too:** mountains, rivers, ...



Transport technology: Endogenous iceberg trade costs

Transporting one unit of good n from j to k requires

$$\tau_{jk}^{n} = \tau_{jk}(Q_{jk}^{n}, I_{jk})$$

units of the good itself, where

- $Q_{jk}^{n} \equiv$ flow of good n along jk link **regardless of where it was produced**
- $I_{jk} \equiv$ level of infrastructure on jk link

We assume that, along all links jk,

$$\underbrace{\frac{\partial \tau_{jk}(Q, I)}{\partial Q} \geq 0}_{\text{congestion}} \quad \text{and} \quad \underbrace{\frac{\partial \tau_{jk}(Q, I)}{\partial I} \leq 0}_{\text{paved, more lanes}},$$

and $\tau_{jk}(Q, I) \neq \tau_{i\ell}(Q, I)$ because of topography (distance, ruggedness, ...)

Conservation of flows constraint and the shadow price of goods

For all locations $j = 1, \ldots, J$ and commodities $n = 1, \ldots, N$,

$$\underbrace{D_j^n + \sum_{n'} X_j^{nn'} + \sum_{k \in \mathcal{N}(j)} (1 + \tau_{jk}^n) Q_{jk}^n}_{\text{Consumption + Intermediate Use + Exports}} \leq \underbrace{Y_j^n + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n}_{\text{Production + Imports}}$$

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Let P_j^n denote the **planner's multiplier** on this constraint

- $P_j^n =$ society's shadow value of marginal unit of good n in location j
- $P_j^n =$ price of good n in location j in efficient CE

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Infrastructure technology

- Define transport network as $\{I_{jk}\}_{j \in \mathcal{J}, k \in \mathcal{N}_j}$
- Infrastructure built with one scarce resource: "asphalt"
    - in fixed aggregate supply K
    - freely shipped across locations
    - only used for building infrastructure
    - all together: **opportunity cost of building I_{jk} is just not building $I_{i\ell}$**

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- $\delta^I_{jk} \equiv$ asphalt intensity of building infrastructure on link jk
- **network-building constraint**: with multiplier P_K ("price of asphalt")

$$\sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{N}(j)} \delta^I_{jk} I_{jk} \leq K$$

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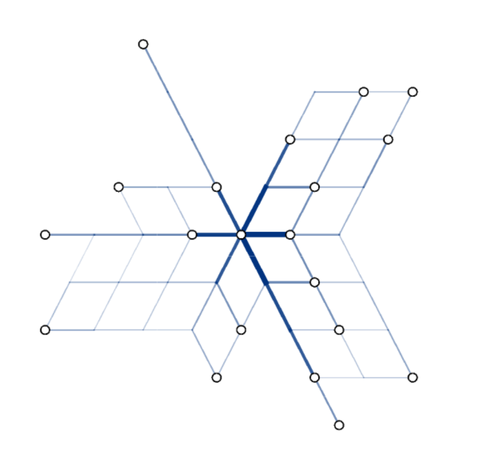
- problem allows for bounds on infrastructure:

$$0 \leq \underbrace{\underline{I}_{jk}}_{\text{existing road}} \leq I_{jk} \leq \underbrace{\overline{I}_{jk}}_{\text{feasible road}} \leq \infty$$

Let's recap with a figure

- neoclassical HH, firms at each node in $\mathcal{J}$



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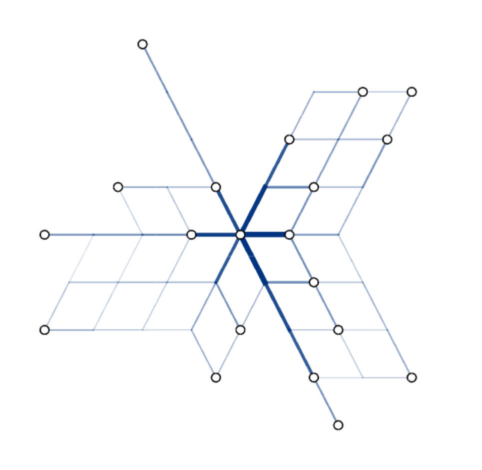
- trade only along **connected** edges in $\mathcal{E}$



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- neoclassical HH, firms at each node in $\mathcal{J}$
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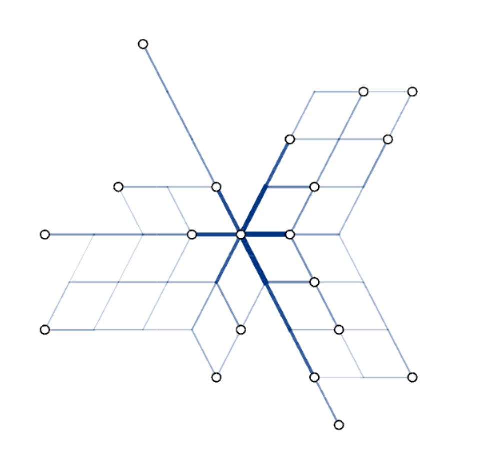
- trade costs τ^n_{jk} **decrease** in level of infrastructure I_{jk} (blue thickness) but **increase** in volume of trade flows Q^n_{jk}



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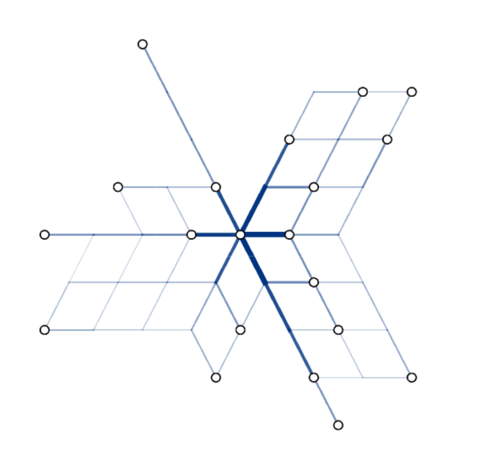
- trade costs τ^n_{jk} **decrease** in level of infrastructure I_{jk} (blue thickness) but **increase** in volume of trade flows Q^n_{jk}
- asphalt K is **scarce** but **sunk**



Planner's problem (with fixed labor)

$$W = \max_{\substack{c_j, h_j, \{I_{jk}\}, \\ \{D_j^n, L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n, \{Q_{jk}^n\}_k\}_n}} \sum_j \omega_j L_j U(c_j, h_j)$$

subject to

1. **availability** of commodities: $c_j L_j \leq D_j(D_j^1, \ldots, D_j^n)$ and $h_j L_j \leq H_j$ for all j
2. **conservation of flows** constraint: for all j, n
$$D_j^n + \sum_{n'} X_j^{nn'} + \sum_{k \in \mathcal{N}(j)} (1 + \tau_{jk}^n) Q_{jk}^n \leq Y_j^n + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n$$
3. **network-building** constraint (subject to bounds on I_{jk}):
$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{N}(j)} \delta_{jk}^I I_{jk} \leq K$$
4. local **factor market clearing** (L_j and $\mathbf{V}_j$)
5. **non-negativitiy** constraints on consumption, flows, and factor use

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$$\boxed{W = \max_{\{I_{jk}\}} \max_{\{Q^n_{jk}\}} \max_{\substack{c_j, h_j \\ \{D^n_j, L^n_j, \mathbf{V}^n_j, \mathbf{X}^n_j\}_n}} \sum_j \omega_j L_j U(c_j, h_j)}$$

subject to

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3. **network-building** constraint (subject to bounds on I_{jk}):

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Convexity of the planner's problem

(i) Given the network $\{I_{jk}\}$, the joint optimal transport and allocation problem (with fixed labor) is a **convex optimization problem** if

$$Q\tau_{jk}(Q, I) \text{ is convex in } Q, \quad \forall j \in \mathcal{J}, k \in \mathcal{N}(j);$$

(ii) the full planner's problem including the network-design problem is a convex optimization problem if, in addition,

$$Q\tau_{jk}(Q, I) \text{ is convex in } Q \textbf{ and } I, \quad \forall j \in \mathcal{J}, k \in \mathcal{N}(j);$$

Intuition: two complementary forces...

- large flows $\implies$ congestion
- large flows $\implies$ incentive to increase capacity

Eventually, congestion must win out for the full problem to be convex

Numerical implementation: Convexity pays off!

If the full planner's problem is convex, then the KKT conditions are **sufficient**, and thus the solution to the **dual problem**

$$\inf_{\mathbf{P}\geq 0} \sup_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \mathbf{P}),$$

coincides with the solution to the **primal problem**

$$\sup_{\mathbf{x}} \inf_{\mathbf{P}\geq 0} \mathcal{L}(\mathbf{x}, \mathbf{P}).$$

(i) use FOCs to express control variables as function of prices, $\mathbf{x}(\mathbf{P})$

(ii) reduce dual to convex minimization problem **just in space of prices**:

$$\inf_{\mathbf{P}\geq 0} \mathcal{L}(\mathbf{x}(\mathbf{P}), \mathbf{P})$$

(iii) solve with gradient-descent algorithm $\rightarrow$ guaranteed to converge to **global** optimum

Optimal flows

Get a **no-arbitrage condition** from the planner's first-order condition for flows:

$$\frac{P_k^n}{P_j^n} \leq 1 + \tau_{jk}^n + \frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n} Q_{jk}^n, \quad = \text{ if } Q_{jk}^n > 0.$$

Any price differential that exceeds the marginal transport cost would be exploited.

- with no congestion ($\partial \tau_{jk}^n / \partial Q_{jk}^n = 0$), price differential bounded by trade cost
- if $Q \tau_{jk}(Q, I)$ is convex in Q, can **invert RHS** to get $\tilde{Q}_{jk}^n(P_k^n / P_j^n)$, which is **increasing**
- each good n flows in only **one direction** (to the higher price)

Optimal network

With upper bound $\bar{I}_{jk}$ not binding, the planner's first-order condition for infrastructure is:

$$\underbrace{P_K \delta^I_{jk}}_{\text{Marginal building cost}} \geq \underbrace{\sum_n P^n_j Q^n_{jk} \left(-\frac{\partial \tau^n_{jk}}{\partial I_{jk}} \right),}_{\text{Marginal gains from infrastructure}} \qquad = \text{ if } I_{jk} > \underline{I}_{jk}.$$

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Workhorse parameterization: Log-linear transport costs

Suppose that

$$\tau_{jk}(Q, I) = \delta_{jk}^{\tau} \frac{Q^{\beta}}{I^{\gamma}}, \quad \text{with } \beta \geq 0, \gamma \geq 0$$

Main result: $Q\tau_{jk}(Q, I)$ **is convex in both arguments iff** $\beta \geq \gamma$

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Also get intuitive, closed-form solutions:

- **optimal flows:**

$$Q_{jk}^{n} = \left[\frac{1}{1+\beta} \frac{I_{jk}^{\gamma}}{\delta_{jk}^{\tau}} \max\left\{ \frac{P_k^n}{P_j^n} - 1, 0 \right\} \right]^{\frac{1}{\beta}}$$

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- **optimal network:**

$$I_{jk} = \min\{\max\{I^*_{jk}, \underline{I}_{jk}\}, \overline{I}_{jk}\}, \quad \text{with } I^*_{jk} = \left[\frac{\gamma}{P_K} \frac{\delta^{\tau}_{jk}}{\delta^{I}_{jk}} \left(\sum_n P^n_j (Q^n_{jk})^{1+\beta} \right) \right]^{\frac{1}{1+\gamma}}$$

Taking the planner's problem to the data

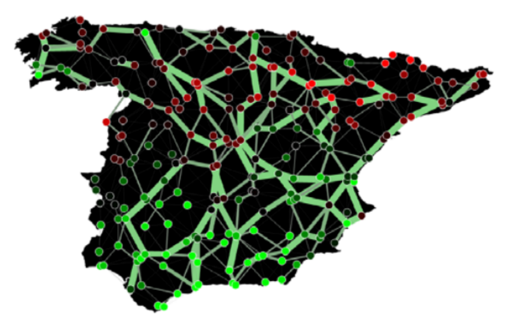
Wait, does that make sense?

Parameterization

- **preferences:** $c_j^{\alpha} h_j^{1-\alpha}$, CES
- **production:** $Y_j^n = z_j^n L_j^n$
- **transport:** log-linear (previous slide)
- **traded sectors:** 11 (10 diff., 1 homog.)

Vars/params	Calibration
$\mathcal{J}$	0.5 arc-degree cells
$\mathcal{E}$	connect contiguous cells
I_{jk}^{obs}	weighted sum of lanes
L_j^{obs}	NASA-SEDAC's GPW v.4
$\text{GDP}_j^{\text{obs}}$	Yale's G-Econ 4.0
z_j^n	model-implied
H_j	model-implied
$\alpha = 0.4$	exp. share of local cons.
$\beta = 0.13$	Couture et al. (2018)
$\gamma = 0.10$	Couture et al. (2018)
δ_{jk}^{τ}	distance_{jk}
δ_{jk}^{I}	distance_{jk}, ruggedness_{jk}

Optimal road expansion and reallocation in Spain

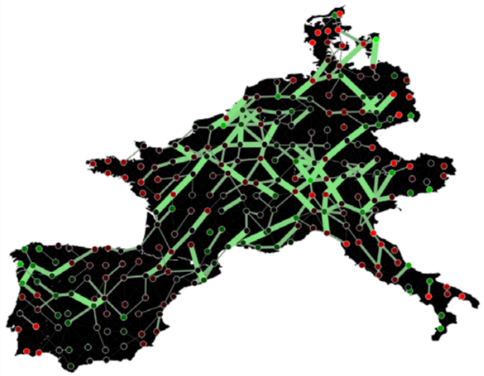


Expand: $1.5K$ with $\underline{I}_{jk} = I_{jk}^{\text{obs}}$

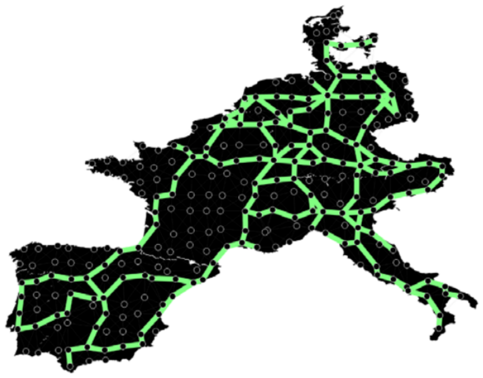
Reallocate: K with $\underline{I}_{jk} = 0$

Same exercise across 24 countries: avg. welfare gain of 1.8% for both

Optimal road expansion across Western Europe vs. TEN-T proposal



Model (labor mobile within countries)



Discretized TEN-T network

total welfare gains of 2.5%

Appendix

The decentralized equilibrium can be characterized, but not matched to data

Full planner's problem can be decentralized **with proper set of Pigouvian taxes**:

- atomistic shippers pay tolls
- link-specific contractors build roads, earn toll revenue
- tolls set to perfectly offset congestion

But that set of Pigouvian taxes is not in the DGP!

- the DGP may be a competitive equilibrium, but it's not this one

So why do it this way? **Anything weaker and we lose global optimality** back