Solutions: Problem Set 1

Economics 27000

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January 25, 2021

Question 1

1. **True.** It's sufficient to notice that the autarky production bundle is always feasible. Free trade only expands the feasible set, which cannot make a country worse off. Of course, as we discussed in class, free trade need not make a country *strictly* better off.

2. **False.** Without trade costs, this would be true (see slide 65). But consider the extreme case wherein trade costs are prohibitively high. So long as there is a strictly positive demand for each good in each country, then all goods will be produced in both countries.

3. **Uncertain.** The simple answer is **true**: outside of knife-edge cases, a uniform increase in Chinese productivity across all goods (which is isomorphic to an increase in the Chinese labor endowment) will improve US terms of trade and increase welfare. Strictly speaking, though, whether or not the statement is true depends on *if* the terms of trade shift. If the initial equilibrium of the 2-by-2 Ricardian model was characterized by incomplete specialization wherein the US produced both goods, then the US wasn't gaining anything from trade. If the uniform increase in Chinese productivity is not large enough to sustain an equilibrium with complete specialization, then the US still gains nothing from trade, so its welfare does not increase. Try playing this out using the diagrams on slides 69–74.

Question 2

Let's establish a bit of notation to help us below. Let $(a_s, a_j) = (1, 2)$ denote the unit labor requirements for hockey sticks and jerseys in the US. Similarly, let $(a_s^*, a_j^*) = (4, 3)$ denote the unit labor requirements for hockey sticks and jerseys in Canada.

1. The US has an absolute advantage in both goods because $a_s < a_s^*$ and $a_j < a_j^*$.

2. Let's compare the relative labor requirements across the two goods in the two countries:

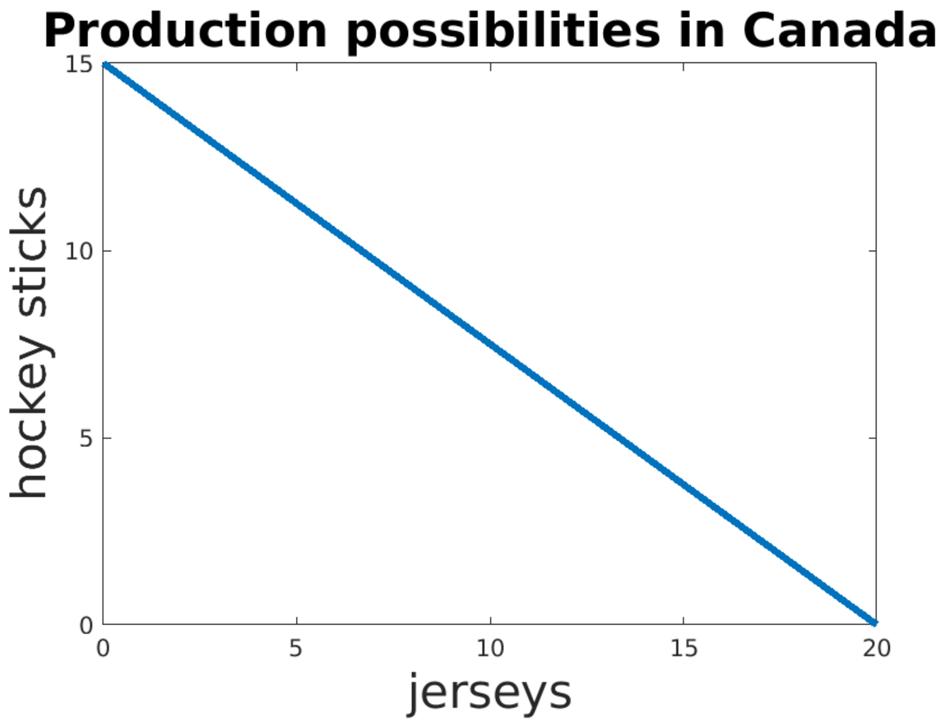
$$\frac{a_s}{a_j} = \frac{1}{2} < \frac{4}{3} = \frac{a_s^*}{a_j^*}. \tag{1}$$

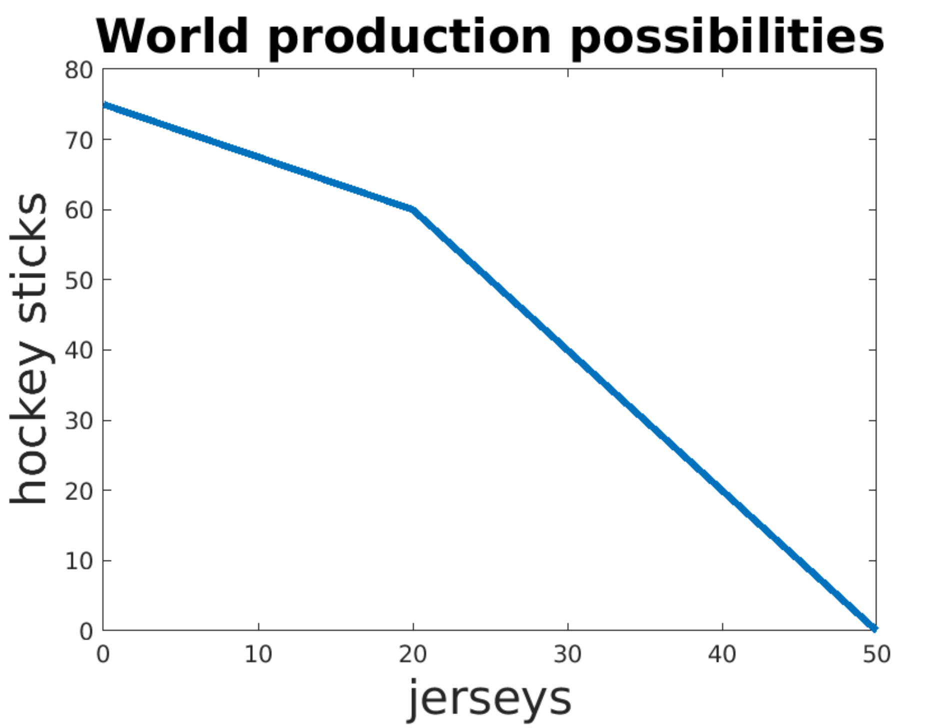
The opportunity cost of one hockey stick in the US is 1/2 of a jersey; in Canada that cost is 4/3 jerseys. Accordingly, the US has a comparative advantage in hockey sticks. The same relationship implies that the opportunity cost of a jersey is lower in Canada than in the US (3/4 of a hockey stick vs. 2 hockey sticks), so Canada has a comparative advantage in jerseys.

3. In 60 hours, the US can produce at most 60 hockey sticks or 30 jerseys at a constant relative price of 2 hockey sticks per jersey. In the same 60 hours, Canada can produce at most 15 hockey sticks or 20 jerseys at a constant relative price of 3/4 of a hockey stick per jersey. With hockey sticks on the y-axis, the slope of the production possibilities frontier for the US is -2; the same slope for Canada is $-3/4$. Each slope indicates the opportunity cost of making a jersey in units of hockey sticks.

4. In 120 total hours, at most 75 hockey sticks or 50 jerseys could be produced worldwide. Because jerseys are cheaper in Canada than in the US (in units of hockey sticks), the first 20 jerseys (Canada's max output) would be produced in Canada. Accordingly, the slope of the world PPF over the range $(0, 20)$ will be $-3/4$, the opportunity cost of making jerseys in Canada. After 20 jerseys, any more must be made in the US at a cost of 2 hockey sticks per jersey. So the slope of the world PPF over the range $[20, 50]$ will be -2. It follows that there will be a kink at the point $(20, 60)$, to the left of which the slope is $-3/4$ and to the right of which the slope is -2.

5. Let's use what we know about Leontief preferences: the consumption bundle that maximizes utility subject to the budget constraint in each country must be one in which jerseys and hockey sticks are consumed in equal amounts: $D_j = D_s$. Accordingly, world production of jerseys must equal world production of hockey sticks. Profit maximization then implies that we seek the point on the world PPF where this is true, namely $(100/3, 100/3)$. At $(100/3, 100/3)$, it must be that Canada has used its entire capacity to produce 20 jerseys – because it had the comparative advantage in jerseys but could not meet the world's demand with its capacity – while the US has produced the remaining 40/3 jerseys plus all 100/3 hockey sticks.


Production possibilities in Canada
hockey sticks
jerseys
0
5
10
15
20


World production possibilities
hockey sticks
jerseys
0
10
20
30
40
50
60
70
80

It remains to determine each country's income and, from there, trade flows. We can normalize one price: let's set $p_s = 1$. At $(100/3, 100/3)$, the slope of the PPF is -2, so

$$-p_j/p_s = -2 \implies p_j = 2$$

in equilibrium. Canadian income from producing 20 jerseys is therefore $I^* = 20p_j = 40$, and the Canadian consumption bundle is $(D_j^*, D_s^*) = (40/3, 40/3)$. US income from producing 40/3 jerseys and 100/3 hockey sticks is $I = 60$, and the US consumption bundle is $(D_j, D_s) = (20, 20)$. From here, computing the pattern of trade is easy: it must be that 40/3 hockey sticks are traded from the US to Canada for 20/3 jerseys.

6. When the US labor force expands, the pattern of specialization is left unchanged: Canada still specializes entirely in jerseys, while the US produces both goods. The terms of trade are left unchanged, too: the relative price of jerseys to hockey sticks is still $p_j/p_s = 2$.

   Before we answer the second part of the question, let's back up. Notice that the US did not gain anything from trade in the initial equilibrium: relative prices with free trade were equal to what relative prices would have been in the US under autarky. (Canada, by contrast, was gaining from trade.) With the expansion of the US labor force, we've just established that the trade pattern and the terms of trade do not change relative to the initial free trade equilibrium. We can deduce, then, that the US still does not gain from trade. Relative to the US with 600 worker hours and free trade with Canada, a counterfactual US with 600 worker hours but in autarky would be no worse off.

Question 3

1. Marginal utility of clothing:

$$MU_C = \frac{\partial U}{\partial D_C} = \alpha \left(\frac{D_F}{D_C}\right)^{1-\alpha}$$

2. Marginal utility of food:

$$MU_F = \frac{\partial U}{\partial D_C} = (1-\alpha) \left(\frac{D_C}{D_F}\right)^{\alpha}$$

3. You just need to remember the definition of the marginal rate of substitution (MRS) and to have done the first two parts correctly. Recall that the MRS is the slope of the indifference curve at a given point. This is the rate at which you would be willing to give up a bit of one good to get a bit of another.

$$MRS = -\frac{MU_C}{MU_F} = -\frac{\alpha D_F}{(1-\alpha)D_C}$$

4. Use the identity $MRS = -P_C/P_F$:

$$-\frac{P_C}{P_F} = -\frac{\alpha D_F}{(1-\alpha)D_C} \quad \Rightarrow \quad D_F = \frac{P_C D_C}{P_F}\frac{1-\alpha}{\alpha}. \tag{2}$$

Substitute this into the budget constraint:

$$\begin{aligned}
I &= P_C D_C + P_F D_F \\
&= P_C D_C = P_F \frac{P_C D_C}{P_F} \frac{1-\alpha}{\alpha} \\
&= P_C D_C (1 + \frac{1-\alpha}{\alpha}) \\
&= \frac{P_C D_C}{\alpha} \\
\Rightarrow D_C^* &= \frac{\alpha I}{P_C}.
\end{aligned}$$

Now substitute D_C^* into the equation for D_F in (2):

$$D_F^* = \frac{(1-\alpha)I}{P_F}.$$

5. Share of income spent on each good:

$$\begin{aligned}
\frac{P_F D_F^*}{I} &= \frac{P_F \left(\frac{(1-\alpha)I}{P_F}\right)}{I} = \frac{(1-\alpha)I}{I} = 1-\alpha, \\
\frac{P_C D_C^*}{I} &= \alpha.
\end{aligned}$$

This is a particular trait of Cobb-Douglas preferences.

Question 4

1. There is no comparative advantage in this case. Every tradable has the same productivity in both countries. Due to the trade costs, then, there will be no trade.

2. Because US productivity in car and electronic production are the same, and because US productivity in services is 3 times higher than in either kind of manufacturing, we have $p_e = 1$ and $p_s = 1/3$. The price of cars in Japan must be $p_c^* = 1$; otherwise, either Japan or the US would import from the other at the cheaper price. It follows by the same logic as above that $p_e^* = 1$ and $p_s^* = 1$.

3. From the zero-profit condition, $p_e = w * \frac{1}{a_e}$ so $w = 1$ and $p_e^* = w^* * \frac{1}{a_e^*}$, so $w^* = 1$, too.

4. Using the Cobb-Douglas preference property, $C_c = 0.25 \times E$, $C_e = 0.25 \times E$, and $C_s = 0.5 \times 3 \times E$. Likewise, $C_c^* = 0.25 \times E^*$, $C_e^* = 0.25 \times E^*$, and $C_s^* = 0.5 \times E^*$.

5. US income is $w \times L = 10$, so the consumption bundle is $(2.5, 2.5, 15)$, which is equal to the production bundle. American utility is $2.5^{0.25} \times 2.5^{0.25} \times 15^{0.5} = 6.124$. On the other hand, the income of Japan is $w^* \times L^* = 5$, so the consumption bundle is $(1.25, 1.25, 2.5)$, which is equal to the production bundle. Japanese utility is $1.25^{0.25} \times 1.25^{0.25} \times 2.5^{0.5} = 1.76777$.

6. (a) Now the US has a comparative advantage in eletronics while Japan has one in cars. We'll guess for now that Japan will produce only cars but that the US will produce both cars and electronics. We'll verify that guess in part (e).

(b) We still set $p_c = 1$, hence $p_e = 1/2$ and $p_s = 1$ by the equilibrium condition

$$p_c/a_c = p_e/a_e = p_s/a_s.$$

This is a combination of three things: (i) the assumption that the US makes both tradables, (ii) the zero-profit condition in each sector, and (iii) the free mobility of labor across sectors (so wages equalize). Since there is trade, the prices of tradable goods are equal in each country, so $p_c^* = 1$ and $p_e^* = 1/2$. That $p_s^* = 1$ then follows again from the analogous equilibrium condition in Japan: $p_c^*/a_c^* = p_s^*/a_s^*$.

(c) Wages are still $w = 1$ and $w^* = 1$ by the same logic as before.

(d) Using the Cobb-Douglas preference property, $C_c = 0.25 \times E$, $C_e = 0.25 \times 2 \times E$, and $C_s = 0.5 \times E$. Likewise, $C_c^* = 0.25 \times E^*$, $C_e^* = 0.25 \times 2 \times E^*$, and $C_s^* = 0.5 \times E^*$

(e) We know $E = w * L = 10$, so the consumption bundle of the US is $(2.5, 5, 5)$. Similarly, $E^* = 5$ so the consumption bundle of Japan is $(1.25, 2.5, 2.5)$. We know that Japan produces only cars so it will produce 2.5 unit of cars, export 1.25 units and import 2.5 unit of electronics. The utility level of the US is $2.5^{0.25} * 5^{0.25} * 5^{0.5} = 4.20448$; the utility level of Japan is $1.25^{0.25} * 2.5^{0.25} * 2.5^{0.5} = 2.10224$. Finally, we can verify our guess by noticing that Japan produces 2.5 units of services, so they must be producing 2.5 units of cars. World consumption of cars is 3.75, though, so the US must be making the other 1.25 units of cars.

7. Note that American utility actually fell in Q6, where the US had a comparative advantage in a manufacturing sector, not the services sector. This demonstrates that having a comparative advantage in non-tradable goods is not necessarily a problem for the US, contrary to the quote.