

## Problem Set 3: Specific-factors model

This problem set is due on Monday, February 8. It must be submitted by 5pm CT.

### Exercise 1

Determine whether the following statements are TRUE, FALSE, or UNCERTAIN **within the context of the specific-factors model we saw in class**, and justify your answer in one paragraph. Please be concise.

1. There always exists a set of transfers that makes everyone weakly better off from free trade.
2. A rise in the relative price of food will benefit workers in the food sector but hurt workers in the cloth sector.
3. An increase in the stock of capital leads to a decrease in the rental rate of both capital and land.

### Exercise 2

For each of the following production functions, check if it satisfies

- constant returns to scale
- (strictly) diminishing returns to each factor.

Note that  $Z > 0$  is a measure of total factor productivity in each of the production functions below.

1. **Linear:**

$$F(K, L) = Z[\alpha K + (1 - \alpha)L], \quad \alpha \in [0, 1].$$

2. **Leontief:**

$$F(K, L) = Z \min\{K, L\}.$$

3. **Cobb-Douglas:**

$$F(K, L) = ZK^\alpha L^{1-\alpha}, \quad \alpha \in [0, 1].$$

4. **Constant Elasticity of Substitution (CES):**

$$F(K, L) = Z[\alpha K^\sigma + (1 - \alpha)L^\sigma]^{1/\sigma}, \quad \sigma \in [-\infty, 1], \quad \alpha \in [0, 1].$$

5. **Stone-Geary:**

$$F(K, L) = Z(K - \bar{K})^\alpha (L - \bar{L})^{1-\alpha}, \quad \bar{K}, \bar{L} > 0, \quad \alpha \in [0, 1].$$

## Exercise 3

Consider the specific-factors model (2 goods, 3 factors, where labor is the flexible factor) for a given country with the following Cobb-Douglas production technologies:

$$\begin{aligned} Q_C &= Z_C K^{1/4} L_C^{3/4} && \text{(cloth production)} \\ Q_F &= Z_F T^{1/4} L_F^{3/4} && \text{(food production)} \end{aligned}$$

where  $Z_C$  and  $Z_F$  are the productivities in both sectors,  $L = L_C + L_F$  is the aggregate endowment of labor,  $K$  is the aggregate endowment of capital, and  $T$  is the aggregate endowment of natural resources.

1. Derive the equation of the (inverse) relative supply curve for this economy ( $p_C/p_F$  as a function of  $Q_C/Q_F$ ). What can you say about the shape of this curve?
2. Suppose that a country opens up to international trade. Assume that this country is small relative to its trading partners, so that the prices of cloth and food are now fixed at world prices:  $p_C = 2$  and  $p_F = 1$ . Further assume that all consumers in this country have Leontief preferences over food and cloth:  $U(Q_C, Q_F) = \min\{Q_C, 4Q_F\}$ . How will the country's pattern of trade depend on the parameters of the model ( $K, L, T, Z_C, Z_F$ )? How does the size of the labor force  $L$  affect the pattern of trade?
3. How will an increase in productivity in the cloth sector ( $Z_C$ ) affect the welfare of all three factor owners? Contrast your response with the welfare change induced by an increase in the aggregate capital stock  $K$ . Assume that world prices remain fixed throughout.